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A. M. Okhrimovskyy, O. V. Podshyvalova

PHYSICS

Guidance Manual
for Laboratory Experiments

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О–92

Посібник містить детальний опис 19 лабораторних робіт з фізики, що пропонуються на кафедрі фізики ХАІ.

Видання складається з трьох частин, які охоплюють основні розділи курсу фізики: механіка і термодинаміка, електрика й магнетизм, хвильова та квантова оптика.

Для англомовних студентів університету.

Reviewers: cand. of techn. science, ass.prof. M. G. Aleksandrov,
cand. of phys.& math.science, ass.prof. O.O. Stepanov

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Manual contains detailed description of 19 physics laboratory experiments offered by the Physics department of the National Aerospace University "Kharkiv Aviation Institute".

Guidance covers three major branches of physics: mechanics and thermodynamics; electricity and magnetism; wave and quantum optics.

For english-speaking students of National Aerospace University "Kharkiv Aviation Institute".

Figs. 46. Tables 15. Bibl.: 7 items

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Contents

INTRODUCTION

The laboratory experiments is one phase of a course of study in physics. It is evident, that there is a greater chance of a student's understanding a topic if it is put before him in more than one way. The course is arranged so that a student meets a given topic in a variety of ways: in reading assignments in the text, in demonstration lectures, in supplementary notes issued to students, in recitation and problem drill in problem sessions, in both the study and the performance of laboratory experiments, in homework problem sets, and in quizzes and examinations. These various types of presentation are synchronized so that, it is hoped, their impact on the student will have a maximum effectiveness.

The laboratory experiments can be an exciting part of the course, or it can be drudgery, depending upon your attitude toward it. If you regard it merely as an impediment to your getting through the course, probably you will not enjoy it, and furthermore, probably you will derive very little benefit from it. On the other hand, if you approach the laboratory with the thought that it is an opportunity to learn, and with a desire to make the most out of it, then it is almost certain you will find the time you spend on it both profitable and interesting.

Most of the principles of physics were discovered by men using equipment no better than yours in fact, most of it was not so good. We hope that you will be able to do some independent thinking about physical principles in the laboratory and that with the equipment in front of you, you will be able to try out your own ideas and find out things for yourself.

Chapter 1

MECHANICS AND THERMODYNAMICS

LABORATORY EXPERIMENT 1.4

MOMENT OF INERTIA OF A FLYWHEEL

Purpose of the Experiment: to estimate the friction force in bearings and the moment of inertia of a flywheel.

Equipment and Accessories: flywheel, block on a thin flexible nonstretchable rope, meter stick, stopwatch, vernier caliper.

Basic Methodology. Inertial lift of a hanging block is lower than its initial height because of the resistance in bearings. Average resistance force can be evaluated on the base of the energy conservation law from the weight of the block and corresponding heights. The highest velocity of the block can be determined from its fall time. Angular velocity of the flywheel is related to the velocity of the block. Moment of inertia of the flywheel can be determined on the base of the energy conservation law.

Recommended Pre-lab Reading: [1] 9.4, 10.2; [2] 9.2, 9.3; [3] 10.4–10.7.

Pre-lab Questions

1. What is the SI unit for the moment of inertia?
2. Provide a definition of a torque.
3. To maximize the moment of inertia while minimizing its weight, what shape and distribution of mass should a flywheel have?

Description of the Equipment

To determine moment of inertia of a flywheel and friction force in bearings, the installation shown in Fig. 1.4.i is used. A flywheel 1 of an unknown moment of inertia I is mounted on a horizontal axle 2. A light rope wrapped around the axle supports a block 3 of mass m. The block does not touch the floor in the equilibrium position (rope totally unwind).

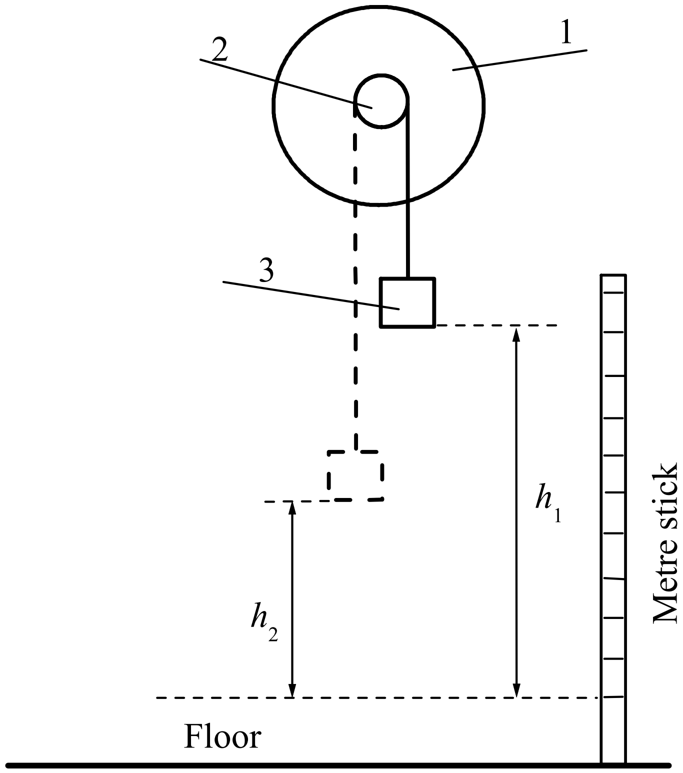


Figure 1.4.i

Theoretical Introduction

A block m is released with no initial velocity at a height h_1 above its equilibrium position. In this case the potential energy of the block is $E_{P1} = mgh_1$. The zero potential energy is assigned to the equilibrium position.

As the block falls, the rope unwinds turning the flywheel. The lowest position of the block corresponds to its highest kinetic energy. Since the rope is considered nonstretchable, the kinetic energy of the flywheel is also the highest. Due to the rotational inertial motion of the flywheel, the rope is wrapped back. As a result, block is lifted to a height $h_2 < h_1$. (Dashed line in the Fig. 1.4.i.) At this instant, the potential energy of the block is equal $E_{P2} = mgh_2$.

The decrease of the potential energy is equal to the work W_f done by the friction force F_f. In this case the energy conservation law would become

$$E_{P1} = E_{P2} + W_f, \tag{1.4.1}$$

where $W_f = F_f(h_1 + h_2)$. In the last expression, we take into account that the total path of the block $h_1 + h_2$ is the same as a displacement of the rim's points against the friction force.

The equation (1.4.3) can be rewritten in terms of heights by substituting expressions for potential energies E_{P1} and E_{P2} in it

$$mgh_1 - mgh_2 = F_f(h_1 + h_2).$$

Previous equation allows us to express the average value of the friction force:

$$F_f = mg\frac{h_1 - h_2}{h_1 + h_2}. \tag{1.4.2}$$

In the lowest position, the block's initial potential energy E_{P1} is converted into the kinetic energy of the system and the work done by a friction force. Total kinetic energy of the system consists of the kinetic energy of the translation motion $\frac{1}{2}mv^2$ of the block and the kinetic energy of rotation motion of the flywheel $\frac{1}{2}J\omega^2$. The work done by the friction force at the way h_1 is

$$W_f = F_f \cdot h_1.$$

According to the energy conservation law

$$mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + F_f \cdot h_1.$$

Previous equation is used to find an expression for a moment of inertia of the flywheel

$$I = \frac{2(mg - F_f) \cdot h_1 - mv^2}{\omega^2}. \tag{1.4.3}$$

Since the block falls under the influence of constant forces, its motion is uniformly accelerated with acceleration a. Taking into account known from the high-school equation $h_1 = \frac{1}{2}at^2$ (where t is a falling time of the block from height h_1), its acceleration would be $a = 2h_1/t^2$. Then, the velocity of the block in the lowest position is

$$v = at = \frac{2h_1}{t}.$$

The speed of the falling block v must be equal to the tangential speed at the outer surface of the axle. Thus, the angular velocity of the flywheel ω is related to v as follows $v = \omega r$, where r is the radius of axle.

After substituting the expression for a friction force F_f from Eq. (1.4.2) into Eq. (1.4.3), the final expression for the moment of inertia of the flywheel can be obtained:

$$I = mr^2 \left[gt^2 \frac{h_2}{h_1(h_1 + h_2)} - 1 \right]. \tag{1.4.4}$$

Step-by-step Procedure of the Experiment

1. Find an equilibrium position of a block m above the floor. It corresponds to the situation when a load hangs freely on a totally unwind rope. Mass of the load is given on it.
2. Wind up the rope tightly on the axle so that the block is lifted to a height h_1 with respect to its initial position. The magnitude of h_1 is provided by a lab instructor.
3. Release the axle and measure the fall time t of the block from height h_1 to its initial position.

Table 1.4.i

#	h_1, m	h_2, m	$\langle h_2 \rangle$, m	t, s	$\langle t \rangle$, s	$\langle r \rangle$, m	F_f, N	I, kg·m^2
1								
2								
3								

4. Measure the height h_2 (above its initial position) of the inertial lift of the block.
5. Repeat steps from 2 through 4 several (3–5) times. Use the same h_1 for all efforts.
6. Measure the diameter d of the axle 2 with a vainer caliper in three different directions. Find an average value of the axle radius $\langle r \rangle$.
7. Record all data to Table 1.4.i.
8. Calculate the average friction force according to Eq. (1.4.2).
9. Apply Eq. (1.4.4) to compute the moment inertia of the flywheel.
10. Analyze obtained results and make conclusions.

After-lab Questions

Case 1.

1. Write down two forms of the basic equation of rotational dynamics.
2. What law is used to derive the basic equation of moment of inertia in this experiment?
3. Based on your experimental data, estimate the highest possible mass of the block that would not cause a rotational motion of the flywheel.
4. *Problem.* Flywheel is rotated with a constant angular velocity by an engine. The power of the engine was turned off. Once started, the flywheel made $N = 120$ revolutions during

$t = 30$ s, and stopped. The moment of inertia of the flywheel equals $I = 0.3$ kg·m^2. The angular acceleration of the flywheel is constant after engine has stopped. Find the power of the engine when a flywheel rotates uniformly.
Answer: $P = 25$ W.

Case 2.

1. Write down formula for calculating moment of inertia of the particle point, disk, and ring.
2. Derive the expression for calculating friction force in this experiment.
3. If the polar ice caps were to completely melt due to global warming, the melted ice would redistribute itself over the earth. How would this change the length of a day (the time needed for the Earth to make one revolution about its axis)?
4. *Problem.* With what force F should you press down the brake block to the wheel which does $f = 30$ rev/s for it to stop in $t = 20$ s? The wheel weights is 10 kg. The weight is distributed over the rim. The diameter d of the wheel is equal to 20 cm. The friction coefficient between the rim and the block is $\mu = 0.5$.
   Answer: $F = 18.84$ N.

Case 3.

1. Give a definition of angular momentum of a body.
2. Derive the expression for calculating moment of inertia of a flywheel in this experiment.
3. You apply equal torques to two different cylinders, one of which has a moment of inertia twice larger than the other. Each cylinder is initially at rest. After one complete rotation, which cylinder has the greater kinetic energy?
4. *Problem.* A wheel with moment of inertia $I = 245$ kg·m^2 is rotating at $f = 20$ rev/s. It comes to rest in $t = 1$ min

if the engine that supports the rotation is turned off. Find the torque of friction force τ_f and number of turnovers N the wheel had done before it stopped?
Answer: $\tau_f = 513$ N·m, $N = 600$.

LABORATORY EXPERIMENT 1.7

MEASURING MOMENT OF INERTIA BY USING A TRIFILAR PENDULUM

Purpose of the Experiment: to measure moment of inertia of a body and to check the parallel-axis theorem.

Equipment and Accessories: trifilar pendulum, stop watch, vernier caliper, investigated bodies.

Basic Methodology. Period of oscillation of a trifilar pendulum depends on the dimensions of the pendulum, mass of the oscillating system, and its moment of inertia. Thus, moment of inertia can be determined by measuring period of oscillation of a trifilar pendulum.

Recommended Pre-lab Reading: [1] 9.5, 10.2; [2] 9.2–9.4; [3] 10.4–10.7.

Pre-lab Questions

1. What is the SI unit for the moment of inertia?
2. Formulate the parallel-axis theorem.
3. Find the moment of inertia of a hoop (a thin-walled hollow ring) with the mass M and radius R about an axis perpendicular to the plane of the hoop at an edge.

Theoretical Introduction

Trifilar pendulum consists of a circular platform of mass m_0 and radius R (see Fig. 1.7.i). The platform is suspended to 3

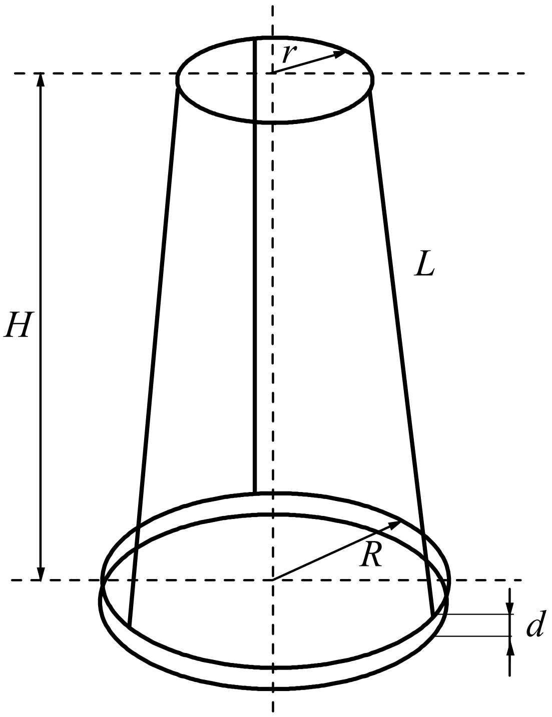


Figure 1.7.i

symmetrical arrangement strings of length L. These strings are symmetrically fixed along the edges of the platform with the lower radius r in the top. Values L, R and r are shown in the equipment.

Top view of the pendulum is shown in Fig. 1.7.ii. If the lower platform is twisted on a certain angle ϕ about its vertical axis, then the horizontal component of a tension force $\vec{\boldsymbol{F}}_{Th}$ of every string would create a restoring torque τ on the platform. The vector of this torque is parallel to the axis of rotation and has a direction opposite to the direction of the twisting angle. According to the definition, the restoring torque is calculated as follows

$$\vec{\tau} = \sum \vec{\boldsymbol{R}} \times \vec{\boldsymbol{F}}_{Th}.$$

Here $\vec{\boldsymbol{R}}$ is a position vector of a knot **B** in a lower platform with respect to the its center **O** (see Fig. 1.7.ii). The section **AB** is a projection of a string on a horizontal plane. **A** is a projection

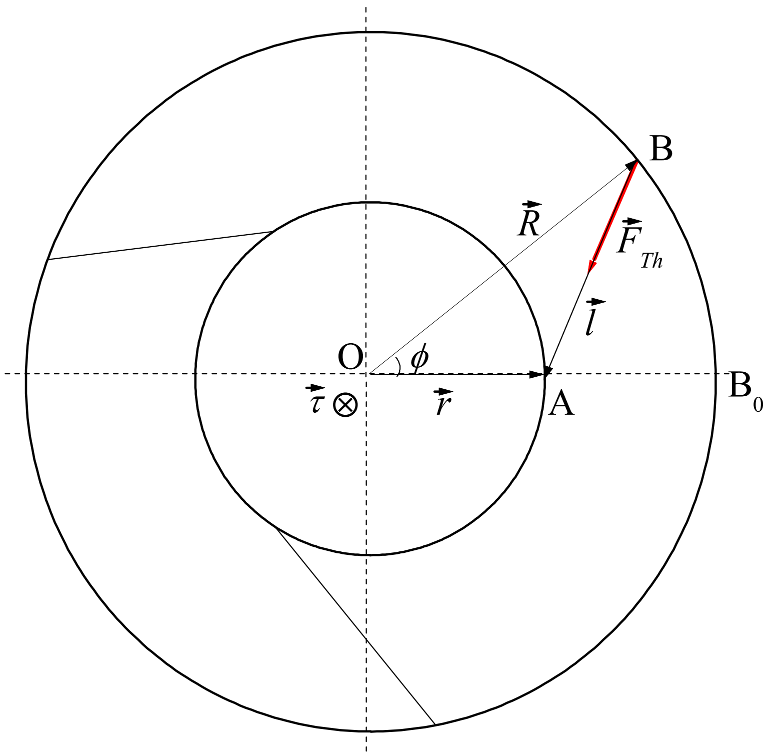


Figure 1.7.ii

of the knot on the fixed platform onto the lower platform. $\vec{\boldsymbol{r}}$ is a position vector of a point **A**.

The horizontal component of a tension force vector $\vec{\boldsymbol{F}}_{Th}$ is directed along the section **AB**. If the length of this section is l, then

$$\vec{\boldsymbol{F}}_{Th} = F_{Th}\frac{\vec{\boldsymbol{l}}}{l},$$

where F_{Th} is a magnitude of this horizontal component.

Since $\vec{\boldsymbol{l}} = \vec{\boldsymbol{r}} - \vec{\boldsymbol{R}}$, torque exerted on a lower platform by one string can be simplified to

$$\vec{\tau}_1 = \vec{\boldsymbol{R}} \times \frac{\vec{\boldsymbol{r}} - \vec{\boldsymbol{R}}}{l} F_{Th} = -\frac{\vec{\boldsymbol{r}} \times \vec{\boldsymbol{R}}}{l} F_{Th}.$$

Projection of the torque onto an axis of rotation is

$$\tau_1 = -\frac{rR}{l} F_{Th} \sin\phi. \tag{1.7.1}$$

To find the value of the horizontal component of the tension force let's, consider the cross section of a pendulum by a vertical plane that contains this string (see Fig. 1.7.iii). If F_T is a magnitude of the tension force that acts along the string, its horizontal projection F_{Th} can be calculated by taking into account the geometrical dimensions of the pendulum.

$$F_{Th} = F_T \frac{l}{L}, \tag{1.7.2}$$

where L is a length of the string. The magnitude of a tension force can be expressed via its vertical component F_{Tv}.

$$F_T = \frac{L}{H} F_{Tv}, \tag{1.7.3}$$

where H is a distance between platforms.

If the length of the string L and the radii of both rotating R and fixed r platforms are known, the height of the pendulum H in the equilibrium can be calculated with the Pythagorean theorem

$$H = \sqrt{L^2 - (R - r)^2}. \tag{1.7.4}$$

If the angle of twist is small, the vertical component of a tension force F_{Tv} is approximately equal to its value in the equilibrium. The latter one is equal to one third of the weight mg of the lower platform (see Fig. 1.7.iii). Thus,

$$F_{Tv} = \frac{1}{3} mg.$$

Consequently substituting this relation into the equations (1.7.3) and (1.7.2) and then in (1.7.1) we can express the magnitude of the torque τ_1 exerted by a single string on the lower platform via measurable values:

$$\tau_1 = -\frac{1}{3} \frac{mgRr}{H} \sin\phi.$$

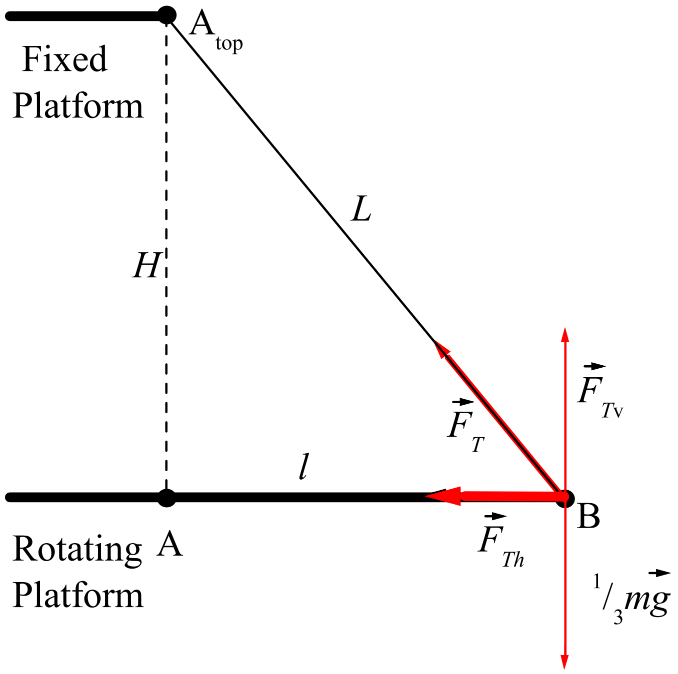


Figure 1.7.iii

The total torque exerted by all three strings is 3 times bigger. According to the main equation of the rotational dynamics,

$$\frac{d^2\phi}{dt^2}I = \tau = -\frac{mgRr}{H}\sin\phi \approx -\frac{mgRr}{H}\phi.$$

In this expression, we took into account that for small angles ϕ its sin is approximately equal to the angle expressed in radians.

Dividing previous equation by the moment of inertia I, we obtain a standard differential equation of the harmonic oscillations:

$$\frac{d^2\phi}{dt^2} = -\omega^2\phi = -\frac{mgRr}{IH}\phi.$$

Thus, the moment of inertia I of the trifilar pendulum can be expressed through the oscillation frequency ω

$$I = \frac{mgRr}{\omega^2 H} = \frac{mgRr}{4\pi^2 H}T^2, \qquad (1.7.5)$$

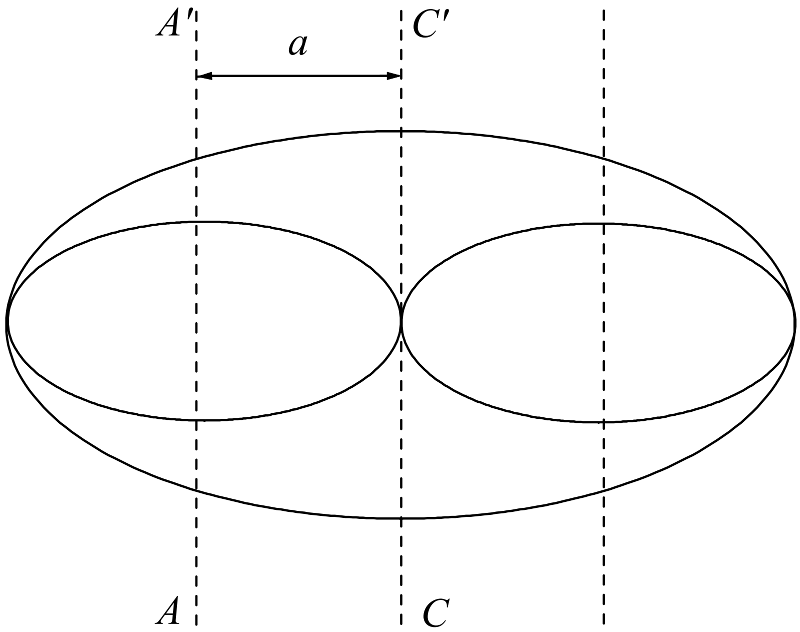


Figure 1.7.iv

where T is a period of oscillations. Relation $T = 2\pi/\omega$ was used in the previous equality.

Moment of inertia of a symmetrical body can be measured with the trifilar pendulum if the axis of rotation of the pendulum passes through the center of mass of the body. If a body placed onto a lower platform participates in the oscillations, then the period of oscillations of the system is different from the period of oscillations of an empty platform. Moment of inertia of a system can be determined according to Eq. (1.7.5) if the total mass of the system is used. To find the moment of inertia of the body, one should employ an additivity properties of it. Thus, if I_0 is a moment of inertia of an empty platform and I_1 is a moment of inertia of the platform with a body, then the moment of inertia of the body I_C about its center of mass is computed as:

$$I_C = I_1 - I_0. \tag{1.7.6}$$

To find the moment of inertia of the body about an arbitrary axis that does not pass through its center of mass, two identical bodies can be placed symmetrically onto the lower platform (see Fig. 1.7.iv). The moment of inertia of the whole system I_2 can be determined with the formula (1.7.5). Then the moment of inertia of two bodies should be calculated with an equation (1.7.6). To find the moment of inertia of a single body, only half of the previous result is required:

$$I_A^{exp} = \frac{I_2 - I_0}{2}. \qquad (1.7.7)$$

If the distance between the axes of oscillations in two cases is a (see Fig. 1.7.iv) and the mass of the body under investigation is m_1, the parallel-axis theorem should be valid:

$$I_A^{th} = I_C + m_1 a^2, \qquad (1.7.8)$$

where I_A^{th} is the moment of inertia of the body about an arbitrary axis AA' and I_C is the moment of inertia of the same body about the axis CC' parallel to AA' and passing through the center of the mass of the body.

Step-by-step Procedure of the Experiment

1. Measure the thickness d of the lower platform by a vernier caliper in several places and find an average value $\langle d \rangle$.
2. Measure the oscillation period T_0 of an empty platform by timing about $N = 30$ small oscillations.
3. Place one of the investigated disks of mass m_1 at the center of the platform and measure the oscillation period T_1 with one body on it.
4. Place both disks on the platform so that they would touch each other at the center of the platform (see Fig. 1.7.iv) and measure the oscillation period T_2 of the trifilar pendulum with two bodies on it.

Table 1.7.i

#	m, kg	N	t, s	T, s	I, kg·m^2
0					
1					
2					

5. Measure the distance a from the symmetry axis of a disk to the axis of the lower platform by a vernier caliper.
6. Compute the mass m_0 of the rotating platform according to the equation $m_0 = \rho V = \rho \pi R^2 \langle d \rangle$, where ρ is the mass density of the lower platform.
7. Estimate the distance H between platforms in equilibrium according to the Pythagorean theorem (1.7.4). Compare obtained value with the length of the string.
8. Calculate the moment of inertia of the empty platform with Eq. (1.7.5). Use m_0 for mass and T_0 for a period of oscillations.
9. Compute the moment of inertia of the platform with one body according to Eq. (1.7.5). Use $m_0 + m_1$ for mass of the pendulum and T_1 for a period of oscillations.
10. Utilize Eq. (1.7.5) to calculate the moment of inertia of the platform with two bodies. In this case, the mass of the oscillating object is $m_0 + 2m_1$, and T_2 is a period of oscillations.
11. Record all data to Table 1.7.i.
12. Use Eq. (1.7.6) to find the moment of inertia of a single disk about the axis passing through its center of mass I_C according to additivity properties of the moment of inertia.
13. Determine the moment of inertia of a single disk about the axis passing through its edge I_A^{exp} taking into account symmetry of the oscillating system with the help of Eq. (1.7.7).
14. Calculate theoretical value of the moment of inertia of the disk about an axis passing through its edge I_A^{th} according to the

parallel-axis theorem (1.7.8). Compare your result with the experimental value I_A^{exp} from the previous step.

15. Analise obtained results and make conclusions.

After-lab Questions

Case 1.

1. Formulate the angular momentum conservation law. Make examples.
2. Derive an expression for a period of oscillations of the trifilar pendulum.
3. A section of a hollow pipe and a solid cylinder of the same radius, mass, and length roll from the same height from the sloped plane. Which object would reach the bottom of the slope faster?
4. *Problem.* A braking wheel reduce the frequency of rotation uniformly from $f_1 = 300$ rpm to $f_2 = 180$ rpm at the time $t = 1$ min. The moment of inertia of the wheel $I = 2$ kg·m^2. Find the retarding torque τ; the braking work W; the number of the revolutions N of the wheel.
   Answer: $\tau = 0.42$ N·m, $N = 240$, $W = 630$ J.

Case 2.

1. Formulate the parallel-axis theorem.
2. Estimate theoretically the period of oscillation of the empty platform in your experiment. Compare the result with your experimental data. Assume the platform is a uniform disk of mass m_0 and radius R.
3. Find the moment of inertia of a uniform disk of mass M and radius R about an axis perpendicular to the disk plane at an edge.
4. *Problem.* Two little bullets of the masses $m_1 = 40$ g and $m_2 = 120$ g are connected by a weightless rod $l = 20$ cm

long. The system rotates about the axis which is perpendicular to the rod and passes through the center of inertia of the system. Determine the angular momentum L of the system about axis of rotation. The rotation frequency $f = 3$ s^{-1}.
Answer: $L = 2.26 \cdot 10^{-2}$ kg·m^2/s.

Case 3.

1. Write down an expression for kinetic energy of the rotating body.
2. Would the period of oscillations of the trifilar pendulum with two disks change in comparison to what you have measured if both disks are placed in the center of the platform one on the top of other?
3. Use the parallel-axis theorem to show that the moments of inertia of a uniform rod about the axis at its center and its edge are consistent with each other.
4. *Problem.* In a homogeneous disk of the mass $m_1 = 1$ kg and radius $R = 30$ cm, a round aperture is cut of the diameter $d = 20$ cm. Its center is at the distance $l = 15$ cm from the axis of the disk. Find the moment of inertia about the axis which passes perpendicular to the surface of the disc through its center.
   Answer: $I = 4.2 \cdot 10^{-2}$ kg·m^2.

LABORATORY EXPERIMENT 1.24

DETERMINATION OF THE COEFFICIENT OF A FLUID INTERNAL FRICTION WITH THE STOKES'S METHOD

Purpose of the Experiment: to investigate phenomenon of internal friction in liquids and to determine the viscosity coefficient.

Equipment and Accessories: tall glass column filled of liquid, small lead and steel spheres, micrometer, stopwatch, ruler.

Basic Methodology. An object moving through a liquid experiences a force in the direction opposite to its motion. Terminal velocity is achieved when the drag force is equal in magnitude but opposite in direction to the force propelling the object. By measuring this velocity, viscosity coefficient can be determined.

Recommended Pre-lab Reading: [1] 5.3, 14.3, 14.6; [2] 5.3, 16.4, 19.7; [3] 5.4, 6.4, 14.4.

Pre-lab Questions

1. What is the SI unit for the dynamic viscosity coefficient? Express it via basic SI units.
2. Formulate the Stokes's law.
3. Formulate the Newton's second law.

Theoretical Introduction

Viscosity is a property of the fluid that indicates the resistance to shear within a fluid. A solid body experiences less resistance if the fluid has a low viscosity. SI unit of dynamic viscosity coefficient is Pa·s.

Viscous resistance of a fluid arises from shear in the velocity profile of flow. If two flat plates have fluid between them, a force is required to move the top one at a constant speed in relation to the bottom one. The force is proportional to the area of the plate and (if the fluid is characterized by a Newtonian viscosity coefficient η) to the relative velocity and inverse distance between plates, i.e., to the velocity gradient $\frac{dv_z}{dx}$.

According to Stokes's law, the magnitude of the resistance force F_d or the drag force experienced by a sphere of diameter d moving in a quiescent viscous fluid of viscosity η with a constant velocity v is governed by the following equation:

$$F_d = 3\pi\eta v d. \tag{1.24.1}$$

Let's consider motion of a solid sphere of mass m with the mass density ρ_s in a fluid with the density ρ_f. A sphere starting from rest in a liquid will experience gravity $\vec{\boldsymbol{F}}_g$ and buoyancy $\vec{\boldsymbol{F}}_b$ forces. Once it begins to move, the drag force $\vec{\boldsymbol{F}}_d$ will act to slow its acceleration (Fig. 1.24.i).

According to the Newton's second law

$$m\frac{d\vec{\boldsymbol{v}}}{dt} = \vec{\boldsymbol{F}}_g + \vec{\boldsymbol{F}}_b + \vec{\boldsymbol{F}}_d. \qquad (1.24.2)$$

The gravity force $\vec{\boldsymbol{F}}_g$ acts downward. The buoyancy $\vec{\boldsymbol{F}}_b$ and drag $\vec{\boldsymbol{F}}_d$ forces act upwards.

The buoyancy force is simply the weight of a displaced fluid. Combining volume of the sphere $V_s = \dfrac{1}{6}\pi d^3$ with the gravitational acceleration g and the density of fluid ρ_f or density of sphere ρ_s one can obtain expressions for calculating buoyancy force

$$F_b = \rho_f g V_s = \frac{1}{6}\pi d^3 \rho_f g$$

and gravity force

$$F_g = \rho_s g V_s = \frac{1}{6}\pi d^3 \rho_s g$$

respectively.

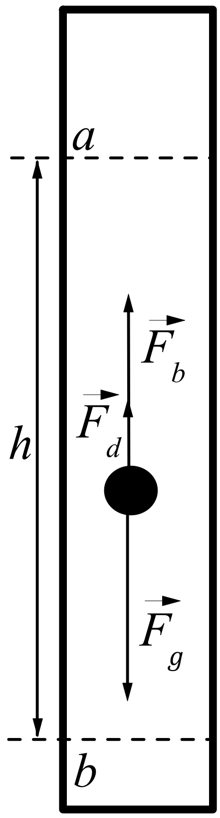


Figure 1.24.i

Substituting force expressions and taking projection of the Eq. (1.24.2) onto the vertical axis directed downward, we obtain the following differential equation:

$$\frac{1}{6}\pi d^3 \rho_s \frac{dv}{dt} = \frac{1}{6}\pi d^3 (\rho_s - \rho_f) g - 3\pi\eta v d.$$

It can be simplified through dividing the left and right sides onto the mass of the sphere $m = \frac{1}{6}\pi d^3 \rho_s$:

$$\frac{dv}{dt} = \frac{\rho_s - \rho_f}{\rho_s} g - \frac{18\eta}{d^2 \rho_s} v. \qquad (1.24.3)$$

If the sphere starts its motion with no initial velocity, the differential equation (1.24.3) has the following solution:

$$v(t) = v_\infty \left(1 - \exp\left(-\frac{t}{\tau}\right)\right), \tag{1.24.4}$$

where v_∞ is the settling velocity and τ is a relaxation time.

The resistance force will increase with velocity. As the sphere moves through fluid, after an initial unsteady transition, it will reach a constant velocity known as the terminal velocity v_∞. At this stage, the left side of Eq. (1.24.2) and (1.24.3) is equal to zero. The sphere would not have an acceleration. Thus, we obtain the expression for terminal velocity

$$v_\infty = \frac{gd^2}{18\eta}\left(\rho_s - \rho_f\right).$$

By reversing this equation, viscosity coefficient can be determined:

$$\eta = \frac{gd^2}{18v_\infty}\left(\rho_s - \rho_f\right). \tag{1.24.5}$$

The relaxation time τ is the time at which the sphere released from rest reaches 63.2% of its terminal speed v_∞. This can be seen by noting that when $t = \tau$ Eq. (1.24.4) yields $v = 0.632\ v_\infty$. Value of the relaxation time is the following

$$\tau = \frac{m}{3\pi\eta d} = \frac{\rho_s d^2}{18\eta} = \frac{\rho_s}{\rho_s - \rho_f}\frac{v_\infty}{g}. \tag{1.24.6}$$

When $t = 3\tau$, velocity of the sphere would reach 95% of its terminal velocity. From that moment, motion is almost settled. To verify that assumption, time dependence of the sphere path $S(t)$ is necessary. This dependence can be found by integrating Eq. (1.24.4) over time

$$S(t) = v_\infty(t - \tau) + v_\infty \tau \exp\left(-\frac{t}{\tau}\right).$$

The exponential term of the previous equation can be neglected if $t \geq 3\tau$. Thus, for $t = 3\tau$ sphere passes the distance $S(3\tau) \approx 2v_\infty\tau$. The sphere should make a pass of $2v_\infty\tau$ through the fluid before it starts to move uniformly.

Reynolds number is a dimensionless parameter that represents the ratio of of inertial forces to viscous forces in fluid.

$$Re = \frac{\rho_f v_\infty d}{\eta}. \tag{1.24.7}$$

Reynolds number provides a good indication of the flow type. Typically all flows can be classified into three categories based on their Reynolds numbers: creeping flow where Reynolds number is very small ($Re \ll 1$) and inertia effects are negligible; laminar flow where flow has a low or intermediate Reynolds number and a smooth varying; turbulent flow where Reynolds number is very high (thousands or even higher) and flow has strong random high-frequency fluctuations.

Step-by-step Procedure of the Experiment

1. Use the micrometer to measure the diameter d of the sphere in three different orientations and find an average value $\langle d \rangle$.
2. Drop the sphere into the fluid. Use the stopwatch to measure time t it takes the sphere to travel from the label **a** to the label **b** (see Fig. 1.24.i).
3. Measure the height of the liquid column h between the labels **a** and **b**.
4. Compute the terminal velocity $v_\infty = h/t$.
5. Calculate the viscosity coefficient η with Eq. (1.24.5).
6. Use equation (1.24.6) to find the relaxation time τ. Calculate the path $2v_\infty\tau$ that the sphere should pass before its speed is settled. Compare it with a height of a liquid column in the cylinder over the label **a** (see Fig. 1.24.i).
7. Repeat steps 1 through 6 with the second sphere.

Table 1.24.i

#	ρ_s, 10^3 kg/m^3	$\langle d \rangle$, 10^{-3} m	t_f, s	v_∞, m/s	η, Pa·s	Re	$\langle \eta \rangle$, Pa·s
1							
2							

8. Calculate the average coefficient of the internal friction of $\langle \eta \rangle$. Record all data to Table 1.24.i.
9. Calculate the Reynolds number Re (eq. 1.24.7).
10. Find the uncertainties (absolute and relative) of the obtained values. Assume that the measured value of the viscosity coefficient η depends on l, t, and d only.
11. Analyze results and make conclusions.

After-lab Questions

Case 1.

1. Why the drop time should be measured only after the sphere passes the label **a** but not at the beginning of the motion?
2. How the terminal velocity v_∞ of the ball would change if its linear size increases?
3. Derive an expression for calculating viscosity coefficient in this experiment.
4. *Problem.* A small sphere of mass 2.00 g is released from rest in a large vessel filled with oil. It experiences a resistive force proportional to its speed. The sphere reaches a terminal speed of 5.00 cm/s. Determine the time constant τ. Ignore the buoyant force.
   Answer: $\tau = 5 \cdot 10^{-3}$ s.

Case 2.

1. What is the relaxation time?
2. Is it possible to conduct this type of experiment with spheres made of wood?

3. Derive an expression for terminal speed in this experiment.
4. *Problem.* A sky diver of mass 80.0 kg jumps from a slow-moving aircraft and reaches the terminal speed of 50.0 m/s. What is the acceleration of the sky diver when the speed is 30.0 m/s?
Answer: $a = 4$ m/s^2.

Case 3.
1. Formulate the Stokes's law.
2. Why the motion of the sphere becomes uniform after certain time?
3. Derive an expression for velocity of the sphere as a function of time.
4. *Problem.* Calculate the force required to pull a iron ball (of density 7.8 g/cm^3) of radius 2.0 cm upward through a fluid at the constant speed 1.0 m/s. Take the drag force proportional to the speed with the proportionality constant equal to 1.0 kg/s. Ignore the buoyant force.
Answer: $F = 3.6$ N.

LABORATORY EXPERIMENT 1.30
HEAT CAPACITIES RATIO MEASUREMENTS

Purpose of the Experiment: to get acquainted with iso-processes in the thermodynamic system and to measure ratio $C_\mathrm{P}/C_\mathrm{V}$ for an air.

Equipment and Accessories: carboy, U-tube pressure gage (manometer), rubber pressure bulb (pump), two valves.

Basic Methodology. Dry air under a small pressure is enclosed in a large vessel having a gas tight valve. The valve is opened for an instant permitting the pressure to become atmospheric and causing the temperature reduction. After the valve is closed, the gas warms up to the room temperature and the pressure increases. From knowing of the initial and final pressures, the ratio of the specific heats is obtained.

Recommended Pre-lab Reading: [1] 19.8; [2] 18.8; [3] 21.2–21.4.

Pre-lab Questions

1. What is the specific heat ratio?
2. What is the adiabatic process?
3. What is the SI unit for specific heat?

Theoretical Introduction

The heat capacity ratio of the gas $\gamma = C_\mathrm{P}/C_\mathrm{V}$ can be determined using the method proposed by Clement and Dezormes. A two step process can be applied to determine this ratio $\gamma = C_\mathrm{P}/C_\mathrm{V}$ experimentally:

- An adiabatic reversible expansion from the initial pressure P_1, to the intermediate pressure, P_0 – atmospheric pressure (dotted curve in Fig. 1.30.i):

$$\text{State } I: \ P_1, V_1, T_1 \Rightarrow \text{State } II: \ P_0, V_2, T_2,$$

  here V_1 and V_2 are specific volumes of a gas (volume per unit mass) in states I and II accordingly.
- Restoration of the temperature to its initial value T_1, at constant volume (solid curve in Fig. 1.30.i):

$$\text{State } II: \ P_0, V_2, T_2 \Rightarrow \text{State } III: \ P_2, V_2, T_1.$$

For adiabatic expansion in states I and II, volume and pressure of an ideal gas are related via Poisson's equation

$$P_1 V_1^\gamma = P_0 V_2^\gamma.$$

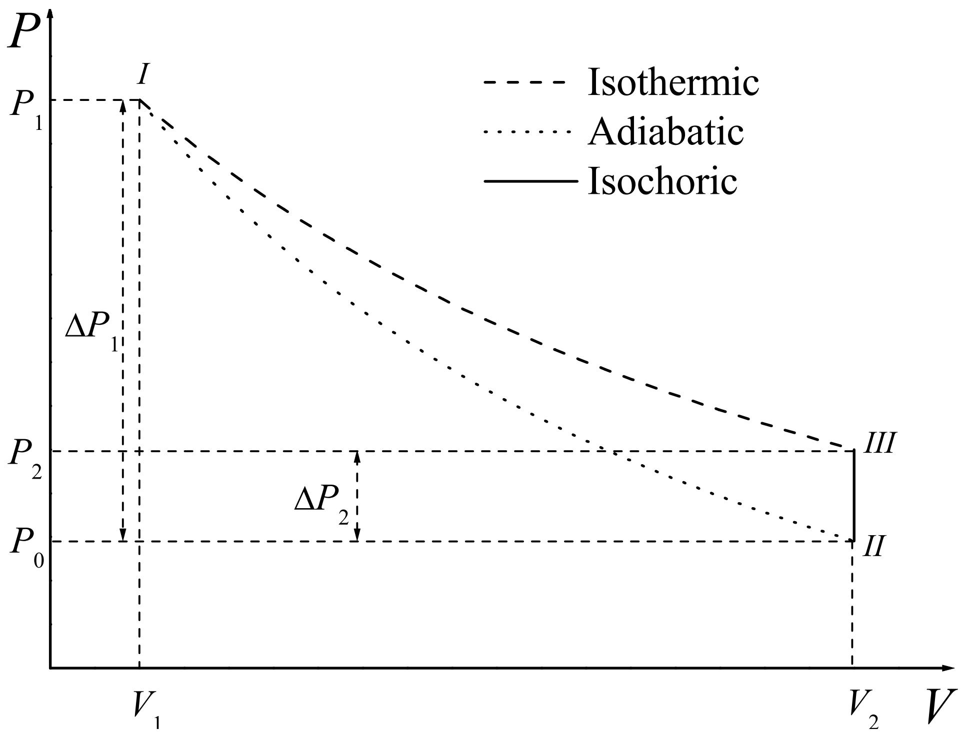


Figure 1.30.i

Thus, γ can be found by solving the previous equation

$$\gamma = \frac{\ln(P_1/P_0)}{\ln(V_2/V_1)}. \tag{1.30.1}$$

The ratio of volumes V_2/V_1 can be expressed via the ratio of pressures P_1/P_2 since states I and III belong to the same isothermal process (dashed curve in Fig. 1.30.i). Thus, volume and pressure at these states have to obey Boyle's law:

$$\frac{V_2}{V_1} = \frac{P_1}{P_2}.$$

Substituting this relation into Eq. (1.30.1), the alternative expression for γ is obtained

$$\gamma = \frac{\ln(P_1/P_0)}{\ln(P_1/P_2)}. \tag{1.30.2}$$

Eq. (1.30.2) is the principal equation of the Clement and Desormes method. If the pressure differences ΔP_1 ($\Delta P_1 = P_1 - P_0$)

and ΔP_2 ($\Delta P_2 = P_2 - P_0$) are small in comparison with the atmospheric pressure P_0, then the expression (1.30.2) can be simplified:

$$\gamma = \frac{\ln\left(1+\frac{\Delta P_1}{P_0}\right)}{\ln\left(1+\frac{\Delta P_1}{P_0}\right) - \ln\left(1+\frac{\Delta P_2}{P_0}\right)} \approx \frac{\frac{\Delta P_1}{P_0}}{\frac{\Delta P_1}{P_0} - \frac{\Delta P_2}{P_0}} = \frac{\Delta P_1}{\Delta P_1 - \Delta P_2}. \tag{1.30.3}$$

Taking into account that extra pressure ΔP is proportional to the difference in height h of the liquid surface in the U-type gage, Eq. (1.30.3) yields to the simplified expression

$$\gamma = \frac{h_1}{h_1 - h_2}, \tag{1.30.4}$$

where $h_1 \sim \Delta P_1$ and $h_2 \sim \Delta P_2$ are heights of the liquid in states I and II accordingly.

The expression (1.30.4) is the main equation used in this experiment to determine the ratio of the specific heats γ with Clement and Desormes method.

Specific heat ratio γ can also be calculated by means of statistical mechanics. The number i of degrees of freedom of the molecule is the number of independent coordinates which must be specified in order to locate the molecule and its component atoms in space.

In statistical mechanics, each degree of freedom contributes, on average, $\frac{1}{2}RT$ of energy per mole of an ideal gas. Thus, for an ideal gas of molecules with i degree of freedom molar heat capacitance under constant volume $C_{\text{V}} = \frac{i}{2}R$. Molar heat capacitance under constant pressure C_{P} (for an ideal gas) can be calculated according to Mayer's relation: $C_{\text{P}} = C_{\text{V}} + R$. Finally, we obtain theoretical expression for the specific heat ratio:

$$\gamma_{th} = \frac{C_{\text{P}}}{C_{\text{V}}} = \frac{\frac{i+2}{2}R}{\frac{i}{2}R} = \frac{i+2}{i} = 1 + \frac{2}{i}. \tag{1.30.5}$$

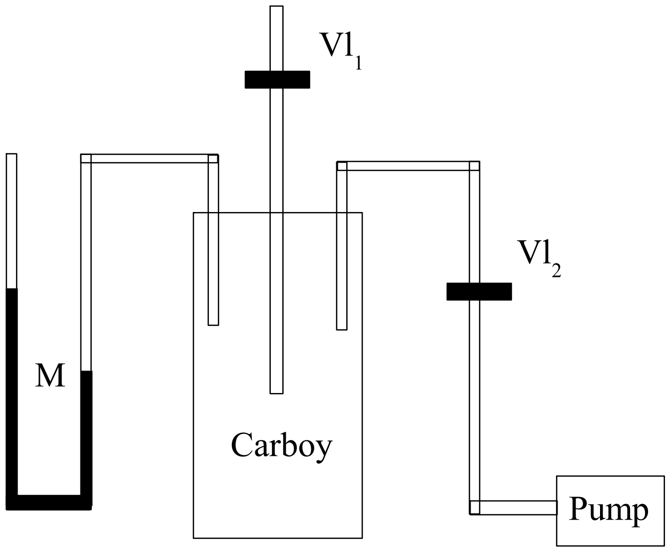


Figure 1.30.ii

Each molecule has at least three translational degrees of freedom. At room temperatures, vibrational degree of freedom can be neglected. A polyatomic molecule also possesses rotational degree of freedom: two for a linear molecules and three for a non-linear one.

Thus, the total number of degrees of freedom i is the sum of three related to the translational motion and those (zero, two or three) related to the rotational motion.

Description of the Equipment

Clement and Desormes method is implemented on an apparatus consisting of a carboy with air, U-tube pressure gage **M** intended for measuring the pressure difference inside and outside the balloon, and rubber pump intended for creating an extra pressure inside the carboy (Fig. 1.30.ii).

To create an initial pressure P_1, the air is pumped up and into the carboy (opened valve $\mathbf{Vl}_2$ and closed $\mathbf{Vl}_1$) until the fluid surfaces of the gage are of 30 – 40 mm different. The air in the carboy is heated under the pressure increase. In 2–3 minutes, the temper-

Table 1.30.i

#	h_1, mm	h_2, mm	γ	$\langle\gamma\rangle$	γ_{th}
1					
2					
3					
4					
5					

ature in the balloon does back to its original value and becomes equal to the room temperature due to heat transfer. After that, the surfaces of the gage get a steady difference in height h_1.

Step-by-step Procedure of the Experiment

1. Close the valve $\mathbf{Vl}_1$. Open the valve $\mathbf{Vl}_2$. Pump extra air into the carboy with a pressure bulb. Close the valve $\mathbf{Vl}_2$. Be sure you have about 50–60 mm level difference on pressure gage.
2. Wait about 5 minutes until the surfaces of the gage have a steady difference in height, and then record the height difference as h_1. **NOTE: If the surfaces are at the same height, there must be some puncture in your device.**
3. Expand the gas adiabatically by opening the valve $\mathbf{Vl}_1$. The air pressure in the carboy will be the same as that in the surroundings. This must be done within 5 seconds. Then close the valve $\mathbf{Vl}_1$ again.
4. Wait about 5 minutes until surfaces of the gage have a steady difference in height, and then record the height difference as h_2.
5. Repeat steps from 1 through 4 five times and record the data to Table 1.30.i.
6. Calculate γ for each run by Eq. (1.30.4). Find the average value $\langle\gamma\rangle$ and its standard deviation.

7. Compute theoretical value γ_{th} for air from number of degrees of freedom according to Eq. (1.30.5). Assume that air consist of molecular oxygen and nitrogen only.
8. Analyze results and make conclusions.

After-lab Questions

Case 1.

1. Formulate the 1st law of thermodynamics.
2. Derive the expression for γ you used in this experiment.
3. If during the experiment the atmospheric pressure changes, would it be legitimate to use Eq. (1.30.4) to calculate γ?
4. *Problem.* Find theoretical values of γ, C_{P}, and C_{V} for methane (CH_4) at room temperatures.

Case 2.

1. Define the internal energy of a thermodynamic system.
2. Derive the Poisson equation for the ideal gas.
3. If the air in the carboy contains water vapor, would the value of γ for this damp air be higher or less than the value of γ for dry air?
4. *Problem.* Find theoretical values of γ, C_{P}, and C_{V} for carbon dioxide (CO_2) at room temperatures.

Case 3.

1. Define the molar heat capacitance and specific heat.
2. Derive the relationship between C_{P} and C_{V} for the ideal gas.
3. Draw a rough graph of the relationship between pressure and volume for isothermal and adiabatic compressions of gas. Which of the two graphs is steeper?
4. *Problem.* Find theoretical values of γ, C_{P}, and C_{V} for ammonia (NH_3) at room temperatures.

LABORATORY EXPERIMENT 1.31
SPEED OF SOUND IN AIR

Purpose of the Experiment: to measure speed of sound in air by exploring resonance effect in standing waves.

Equipment and Accessories: sound generator, loudspeaker, glass tube with air/water column, water tank, flexible pipe, meter stick.

Basic Methodology. The length of the air column in a vertical tube is adjusted by adding and/or removing water from the tube. Notes of various frequencies are sent into the air column. In a pipe closed at one end, resonance can be clearly heard at a certain water level. Distance between consecutive resonance points is a one-half of a wavelength. The speed of sound can be calculated based on the input frequency.

Recommended Pre-lab Reading: [1] 16.2, 16.4; [2] 14.6, 14.7; [3] 17.2, 18.5.

Pre-lab Questions

1. What is a relation between wavelength, frequency of oscillations, and propagation speed of waves?
2. What is a theoretical value of a specific heat ratio for air?
3. Specify acoustic frequency range.

Theoretical Introduction

The mathematical equation for an ideal gas undergoing a reversible adiabatic process (often called Poisson equation) is

$$PV^{\gamma} = \text{const}, \tag{1.31.1}$$

where P is pressure, V is specific or molar volume, and γ is the adiabatic index. Exponent in a Poisson equation is determined as follows:

$$\gamma = \frac{C_\text{P}}{C_\text{V}}, \tag{1.31.2}$$

where C_P being the molar heat capacity of a gas at constant pressure, and C_V being the molar heat capacity of a gas at constant volume.

Adiabatic index can be experimentally determined by standing sound waves.

In gases and liquids sound waves can be only longitudinal. They consist of the alternating compression and depression of the medium (in solid bodies, both longitudinal and transversal waves can spread). Sound oscillations occur so fast that we consider compressions or depressions as the adiabatic.

In small volumes (in comparison with the wavelength), the changes of a gas state are described by Eq. (1.31.1).

After differentiating Eq. (1.31.1), we obtain

$$V^\gamma dP + \gamma P V^{\gamma-1} dV = 0.$$

Whence

$$\frac{dP}{dV} = -\gamma \frac{P}{V}. \tag{1.31.3}$$

If P, V, and γ are positive, then it follows from Eq. (1.31.3) that $dP/dV < 0$ (when pressure increases ($dP > 0$) the volume occupied by the matter decreases ($dV < 0$)).

The propagation speed of the longitudinal waves in an elastic medium is determined by the equation

$$v_s = \sqrt{\frac{dP}{d\rho}}, \tag{1.31.4}$$

where ρ is the mass density of the media. The derivative $\dfrac{dP}{d\rho}$ should be taken under adiabatic condition.

For the speed of sound waves in gas, we obtain

$$v_s = \sqrt{\frac{\gamma P}{\rho}}. \tag{1.31.5}$$

The gas density ρ is determined by the equation of state $PV = \frac{m}{\mu} RT$. As we know, $\rho = m/V$, and

$$\rho = \frac{\mu P}{RT}, \tag{1.31.6}$$

where μ is a molar mass of the gas, T is the absolute temperature of the gas, and $R = 8.314472$ J/(mol·K) is the universal gas constant. Substituting this expression into (1.31.5), we get

$$v_s = \sqrt{\frac{\gamma RT}{\mu}}. \tag{1.31.7}$$

Thus for a given gas, the speed of sound wave propagation is directly proportional to a square root of the absolute temperature, and it does not depend on the pressure of gas.

Velocity of sound in different gases taken under the same conditions is inversely proportional to a square root of their molecular masses. So, the speed of sound in hydrogen ($v_{H_2} = 1263$ m/s, at 273 K) exceeds the speed of sound in air ($v_{air} = 331$ m/s, at 273 K) almost four times.

The ratio between heat capacities γ is deduced from the expression of the speed of sound (1.31.7):

$$\gamma = v_s^2 \frac{\mu}{RT}. \tag{1.31.8}$$

To obtain the numerical value of γ, we measure an absolute temperature of air. The air molar mass is $\mu = 29 \cdot 10^{-3}$ kg/mole.

The speed of sound is determined by the method of standing waves (*the method of Kund*). Two overlapping plane waves which

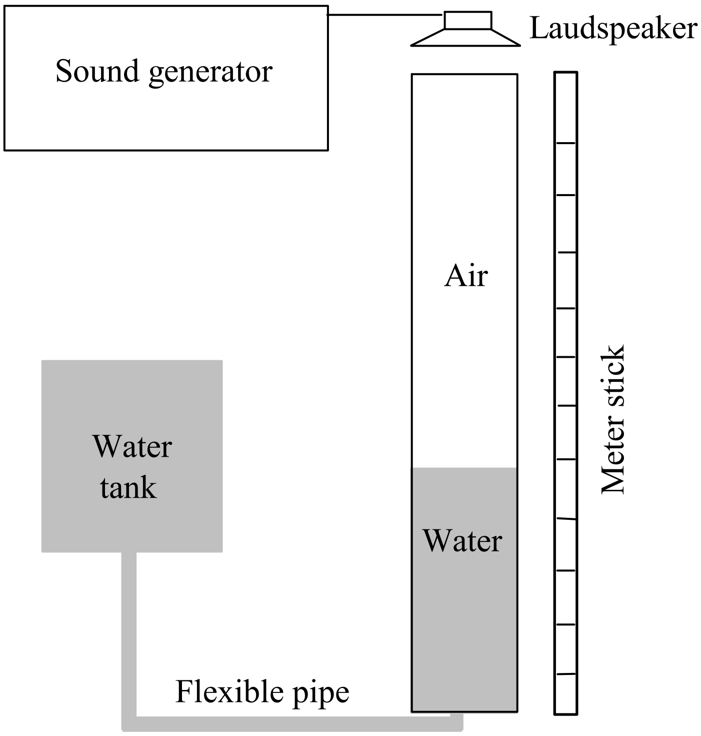


Figure 1.31.i

have the same frequency and amplitude form standing waves. This is the case of interference of the incident and rejected waves.

The installation intended for studying formation of standing waves is shown in Fig. 1.31.i. It serves for measuring the speed of sound in the air v_s. This method is based on the resonance phenomenon. A glass cylinder is joined to the *water tank* by a rubber pipe. There is a *loudspeaker* at the open edge of the cylinder. The diaphragm of the *loudspeaker* is oscillating with a certain frequency provided by the *sound generator*.

In this experiment, longitudinal standing waves are created in a tube containing air. By adjusting the amount of water in the tube, one can lengthen or shorten the column of air in the tube. In tubes, pipes, or columns open at one end and closed at the other, a stable standing wave pattern requires a displacement antinode exist at the open end and a displacement node at the closed end of the tube. This means that the fundamental (first harmonic) standing wave in such a tube occurs when the column of air is of length $\lambda/4$. Here λ is a wavelength. The series of nodes and antinodes form an odd harmonic series in a tube open at one end and closed at the other. Thus, the condition for a standing wave to be formed in a tube closed at one end is

$$h_n = (2n+1)\frac{\lambda}{4}, \tag{1.31.9}$$

where h_n the water level position that corresponds to the n-th resonance. The distance between successive antinodes is Δh calculated as follows:

$$\Delta h = h_{n+1} - h_n = [2(n+1)+1]\frac{\lambda}{4} - (2n+1)\frac{\lambda}{4} = \frac{\lambda}{2}. \tag{1.31.10}$$

To find the speed of sound we should know the frequency of sound f set by the generator and measure Δh:

$$v_s = \lambda f = 2\Delta h f. \tag{1.31.11}$$

Step-by-step Procedure of the Experiment

1. Turn on the power of the sound generator. Set up the output frequency f recommended by your lab instructor. Set the quiet sounding of the dynamic loudspeaker by the handhold. Adjust the output volume to provide a sound at approximately one half of its maximum value.
2. Change the height of the water column by lifting or lowering the water tank. The sound intensity will change. Read positions of water level h_i that correspond to the maximum sound intensity (resonance achieved).
3. Calculate the distance Δh_i for several pairs of consecutive resonance water levels: $\Delta h_i = h_{i+1} - h_i$. Find an average value $\langle \Delta h \rangle$.

Table 1.31.i

#	f, Hz	T, K	Δh_1, m	Δh_2, m	$\langle \Delta h \rangle$, m	v_s, m/s	γ
1							
2							

4. Check the air temperature in the laboratory. Do not forget to convert it into Kelvins, T, K.
5. Record the experimental data f, Δh, T to Table 1.31.i and calculate the speed of sound v_s with Eq. (1.31.11).
6. Calculate the adiabatic index $\gamma = C_\mathrm{P}/C_\mathrm{V}$ according to equation (1.31.8). Compare obtained values with the theoretical γ_{th} given by Eq. (1.30.5). Refer to the comments to that equation in the description of the experiment 1.30.
7. Repeat steps from 1 through 6 with another frequency of sound.
8. Find uncertainties of your measurements.
9. Analyze results and make conclusions.

After-lab Questions

Case 1.

1. What process is called the adiabatic one? Derive the Poisson's equation.
2. Write an expression for the root mean square velocity of gas molecules. Compare it with the expression for calculating the speed of a sound (1.31.7).
3. What is the standing wave?
4. *Problem.* The wave equation for a particular wave is $y(x,t) = 4\sin(\pi(x - 400t)/2)$. All values are in appropriate SI units. What is the amplitude A, wavelength λ, frequency f, and propagation speed v_s of the wave?

Case 2.

1. Give the definition of heat capacities C_P and C_V. Derive the dependence between them.

2. Derive the expression for the speed of a sound in a gas.
3. Explain the reason for changing a sound intensity in the cylinder filled with air that is observed during the experiment.
4. *Problem.* The wave equation for a particular wave is $y(x,t) = 6\cos(\pi(2x - 300t))$. All values are in appropriate SI units. What is the amplitude A, wavenumber k, period T, and propagation speed v_s of the wave?

Case 3.

1. Calculate the adiabatic index for helium, molecular hydrogen, and acetylene (C_2H_2) at room temperatures.
2. How is the wavelength related to the frequency and the propagation speed of sound in the medium?
3. How does the speed of a sound wave depend on the pressure and temperature of the gas?
4. *Problem.* The wave equation for a particular wave is $y(x,t) = 4\cos(\pi(200t - 7x)/2)$. All values are in appropriate SI units. What is the amplitude A, wavelength λ, period T, and propagation speed v of the wave?

Chapter 2

ELECTRICITY AND MAGNETISM

LABORATORY EXPERIMENT 2.1

INVESTIGATION OF THE ELECTROSTATIC FIELD BY THE MODELING METHOD

Purpose of the Experiment: to study an electrostatic field generated by differently shaped electrodes, construction of equipotentials, and lines of an electric field.

Equipment and Accessories: electrolytic bath with differently shaped electrodes and grid; oscilloscope, voltmeter, microammeter, probes, source of AC voltage.

Basic Methodology. The method is based on the similarity demonstrated by electric fields in vacuum and homogeneous electrolyte if they are created by the same system of charges. There are charge carriers in the electrolyte. At every point of the electrolyte current density is proportional to the electric field $\vec{\boldsymbol{E}}$. Measuring the magnitude of the current through a small flat probe, placed at different points of the electrolyte, one can draw electric field vectors.

Recommended Pre-lab Reading: [1] 21.4, 21.6, 23.2, 23.4, 23.5; [2] 22.1, 22.2, 24.2, 24.3, 24.4; [3] 23.4, 23.6, 25.1.

Pre-lab Questions

1. Formulate Coulomb's law.
2. Define an electric field $\vec{\boldsymbol{E}}$.
3. Define a potential.

Theoretical Introduction

An electrostatic field is characterized at every point by the electric field vector $\vec{\boldsymbol{E}}$ and potential φ which are related to each other as

$$\vec{\boldsymbol{E}} = -\operatorname{grad}\varphi. \tag{2.1.1}$$

This field can be graphically represented by electric field lines and equipotential lines. As the electric field lines and the equipotential lines are perpendicular, the electrostatic field can be represented using only field or equipotential lines.

To investigate field and equipotential lines arrangement, in this experiment the method of field modeling with a help of an electrolytic bath is used. The method is based on the similarity demonstrated by fields in vacuum and homogeneous electrolyte when created by the same system of charges. There are charge carriers in electrolyte unlike vacuum or insulator, so to maintain constant potentials on the electrodes a current source should be used. Investigated electric field causes the motion of free charges, i.e. there is an electrical current with density $\vec{\boldsymbol{j}}$. According to Ohm's law,

$$\vec{\boldsymbol{j}}(x, y, z) = \sigma\vec{\boldsymbol{E}}(x, y, z), \tag{2.1.2}$$

where σ is the conductivity of the electrolyte.

At each point of the electrolyte, the current density is proportional to the electric field strength $\vec{\boldsymbol{E}}$, so in the bath (under the condition of homogeneity of the electrolyte), there is formed the field of current density (spatial distribution of current density $\vec{\boldsymbol{j}} = \vec{\boldsymbol{j}}(x, y, z)$), which is similar to the electric field. To investigate the field of current density is much easier than the electric field in vacuum. Current lines can be determined by measuring current strength that passes through some flat area (a small flat probe) which is placed at different points of the electrolyte.

Distribution of potentials can be found with a cylindrical metal electrode (probe). The probe potential is equal to the potential of the point of the field in which the probe is placed. Pay attention that the electric circuit of the probe should have resistance greater than the resistance between the points in the electrolyte, and the probe should have a small size in comparison with the electrodes, otherwise, the presence of the probe can distort the field. To prevent polarization of the electrolyte an AC voltage source of frequency 50 Hz is used. Under such frequency the distribution of currents in the electrolytic bath can be regarded as constant at a moment of time (quasi-stationary).

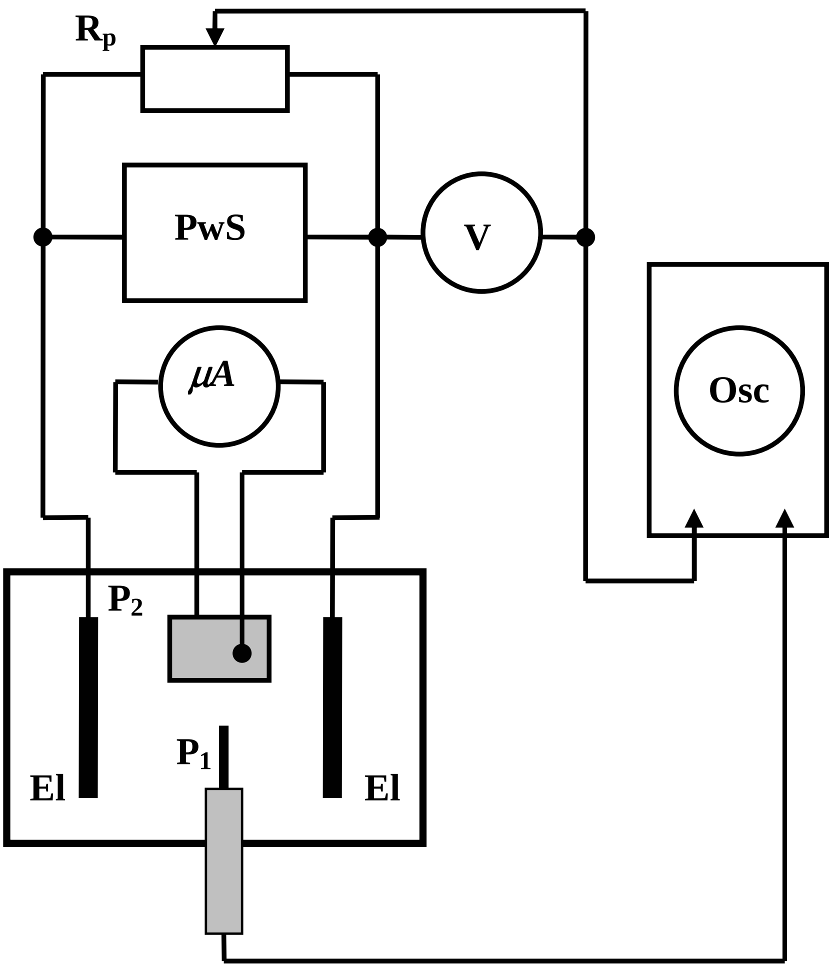


Figure 2.1.i

The experimental setup is shown in Fig. 2.1.i. Metal electrodes **El** are placed in the dielectric bath filled with electrolyte (water).

Power supply **PwS** creates a potential difference at the electrodes, the field between which we want to study.

Position of equipotential lines is determined with help of a cylindrical probe $\mathbf{P}_1$. The probe $\mathbf{P}_1$ and the movable contact of the potentiometer **R** are connected to the oscilloscope (the horizontal scanning of the oscilloscope is off). The length of a vertical line on the oscilloscope screen is proportional to the potential difference between the movable contact of the potentiometer **R** and the probe $\mathbf{P}_1$. If this potential difference is equal to zero, then there is a flashing dot on the screen. Changing position of the probe $\mathbf{P}_1$, you can find a point in the bath in which the potential is equal to the potential of the rolling contact of the potentiometer **R**. These points lie on the equipotential line of the investigated field. A potential of the field on this line is determined by a voltmeter **V**. Changing the position of the rolling contact of the potentiometer **R**, we can determine the position of equipotential lines which have different potentials.

Field lines arrangement, coinciding with the current density lines, is determined with help of the flat probe $\mathbf{P}_2$ and the microammeter. The probe $\mathbf{P}_2$ consists of two metal plates separated by a dielectric layer and connected through a microammeter (see Fig. 2.1.i). To have the current between the plates passing through the microammeter and not through the electrolyte, microammeter resistance must be considerably smaller than that of the electrolyte between the points where the plates are. If the plates are parallel to current density lines (current lines do not pass through the plates), then current does not flow through the microammeter. In this case, electric field lines are also parallel to the plane of the plates. Rotating the probe about the vertical axis, we can obtain the current flowing through the ammeter to be maximal. In this case, the plane of the plates is perpendicular to current density lines and accordingly electric field lines.

Step-by-step Procedure of the Experiment

Task 1. Construction of equipotential lines.

1. On a graph paper (keeping the scale) draw a system of electrodes (the shape of the electrodes is given by an instructor).
2. Fill the bath with water. Water is nearly 10 mm high. Switch on the power supply.
3. Switch on the oscilloscope and warm it up for 2–3 minutes. Turn off the horizontal scanning and, using tuning knobs, locate the electron beam in the center of the screen.
4. Determine the potentials of the electrodes which are specified by power supply. For this, set the handle of the potentiometer $\mathbf{R}$ to the leftmost position and touch one of the electrodes with the probe $\mathbf{P}_1$. There will be a line or a dot on the oscilloscope screen. If there is a dot, the probe potential is zero. If there is a line, the electrode potential is nonzero. To determine the potential of this electrode, rotate the knob of the potentiometer without removing the probe from the electrode until the line on the screen becomes a dot. The voltmeter shows the potential of this electrode.
5. Set the voltage 2 V on the voltmeter. Moving the probe $\mathbf{P}_1$ along the bottom of the bath, find the position when the vertical line on the oscilloscope screen becomes a dot. This position of the probe mark on the graph paper with diagram of the electrodes. Moving the probe in the space between the electrodes with the step of 10 mm, find 5–6 such points and connect them. Received line is an equipotential line of a given potential.
6. Increasing voltage, draw 3–5 equipotential lines as it is described in step 5.

7. Draw field lines of the investigated field considering that they are perpendicular to equipotential lines.
8. Make conclusions.

Task 2. Construction of electric field lines.

1. Place flat probe $\mathbf{P}_2$ in the electrolyte and rotate it about its axis until the current passing through the micro-ammeter becomes zero. In this case, the plane of the electrode is a tangent to the electric field line at a given point of the electrolyte.
2. Find the direction of field lines moving the probe from one electrode to another and along the electrodes with the step of 10 mm.
3. Draw a map of electric field lines. Compare these lines with the lines constructed in Task 1, step 7.
4. Make a conclusion.

Task 3. Check of Gauss law for the electric field.

1. Choose a closed surface in the electrolyte between the electrodes through which the flux of $\vec{\boldsymbol{E}}$ will be determined (a recommended surface is the one which projection on the bath bottom is a square-shaped contour with sides of two widths of the probe $\mathbf{P}_2$).
2. Set the probe $\mathbf{P}_2$ so that the same side faces inwards the surface, and measure the magnitude of the current.
3. Move the probe so that its successive positions draw a closed surface, and measure the magnitude of the current for each probe position.
4. Determine electric flux Φ through a closed surface, using the expression $\Phi = \rho I$ where I is an algebraic sum of the currents, $\rho = 10^3$ Om·m.
5. Make conclusions.

After-lab Questions

Case 1.

1. What is an electric field vector and a potential of the electrostatic field? What is the relationship between them?
2. Formulate the superposition principle of electrostatic fields.
3. What charge distribution should be for reasonable application of the Gauss's law in the integral form to find an electrostatic field?
4. *Problem.* The electric field is given $\vec{\boldsymbol{E}} = -(a\vec{\boldsymbol{\imath}} + b\vec{\boldsymbol{\jmath}} + c\vec{\boldsymbol{k}})$, where a, b, c are constants. Find the potential $\varphi(x, y, z)$ in consideration of $\varphi(0, 0, 0) = 0$.
   Answer: $\varphi = ax + by + cz$.

Case 2.

1. What is an electric flux? Formulate the Gauss's law.
2. Explain why the field lines and equipotential surfaces are mutually perpendicular.
3. Explain the method of modeling in the electrolytic bath.
4. *Problem.* Two like point charges $q_1 = 100$ nC and $q_2 = 200$ nC are at the distance of 1 m from each other. Find the x coordinate of the point at which the electric field produced by these charges is zero. Place the charge q_1 at the origin.

   Answer: $x = 0.41$ m.

Case 3.

1. What is the circulation of electric field equal to? Formulate the condition of field potentiality.
2. Draw the field lines and equipotential surfaces of the field produced by: a) a point charge, b) a uniformly charged plane.
3. By using the Gauss's law, calculate the electric field of a sphere with radius R if the charge is distributed over the surface with density σ.

4. *Problem.* The electric field is given in the form $\varphi = ax^3 + by^2 + cz$, where a, b, c are constant values. Find the dependence of electric field vector on the coordinates $\vec{\boldsymbol{E}}(x, y, z)$.
   Answer: $\vec{\boldsymbol{E}}(x, y, z) = -(3ax^2\vec{\boldsymbol{i}} + 2by\vec{\boldsymbol{j}} + c\vec{\boldsymbol{k}})$.

LABORATORY EXPERIMENT 2.3

DETERMINATION OF THE CAPACITANCE OF A CAPACITOR USING A BALLISTIC GALVANOMETER

Purpose of the Experiment: to determine the capacitance of a capacitor and the battery of capacitors using a ballistic galvanometer.

Equipment and Accessories: power supply of direct current; potentiometer; voltmeter; commutator; reflecting ballistic galvanometer; kit of the capacitors; leads.

Basic Methodology. A capacitor is charged by a power supply and discharged through a ballistic galvanometer. The deflection of the galvanometer light spot is in proportion to the amount of charge that passes through the galvanometer. At the same potential difference across the plates of capacitors, the amount of charge stored on a capacitor is directly proportional to its capacitance. Thus, the capacitances can be determined by discharging capacitors through the ballistic galvanometer.

Recommended Pre-lab Reading: [1] 24.1–24.3; [2] 25.1, 25.2, 25.4; [3] 26.1– 26.4.

Pre-lab Questions

1. What is the capacitance of a capacitor?
2. What is the SI unit for capacitance?
3. Does the value of the capacitance depend on the charge of capacitor, potential difference across its plates? What does it

depend on?

Theoretical Introduction

The ratio of the charge on capacitor plates q to the potential difference between the plates U is called the capacitance C of the capacitor:

$$C = \frac{q}{U}. \tag{2.3.1}$$

The value of the capacitance depends only on the shapes and sizes of the conductors (plates) and on the nature of the insulating material between them.

In this experiment, we determine an unknown capacitance of capacitor using the capacitor with known capacitance. If known capacitance is C_0, and both capacitors have the same potential difference $U_0 = U_1$, then the capacitance of the second capacitor is

$$C_1 = C_0 \frac{q_1}{q_0}. \tag{2.3.2}$$

Thus, to determine the unknown capacity C_1, we have to measure the amount of its charge q_1 and the charge q_0 of the capacitor with known capacitance.

For capacitor charge estimation, we use a ballistic galvanometer. The ballistic galvanometer differs from other moving-coil galvanometers in the way that the moment of inertia of its moving coil is increased by loading the coil. It causes that the period of oscillation of the moving coil is large compared to the time for the charge passing through it. Thus, when the charge passes through it, the coil receives a kick due to an impulse torque. Subsequently, the coil oscillates freely due to the restoring torque provided by the suspension. The maximum deflection amplitude φ (called first throw position) is reached long after the charge has passed. The first throw position φ is proportional to the total charge passing through the coil:

$$q = D\varphi,$$

where D is a proportionality coefficient.

The first throw position φ determines the deflection n (in scale divisions) of the galvanometer light spot. Clearly, n is also directly proportional to the charge q discharged through the ballistic galvanometer:

$$q = Bn, \tag{2.3.3}$$

where B is the coefficient of proportionality called the galvanometer constant.

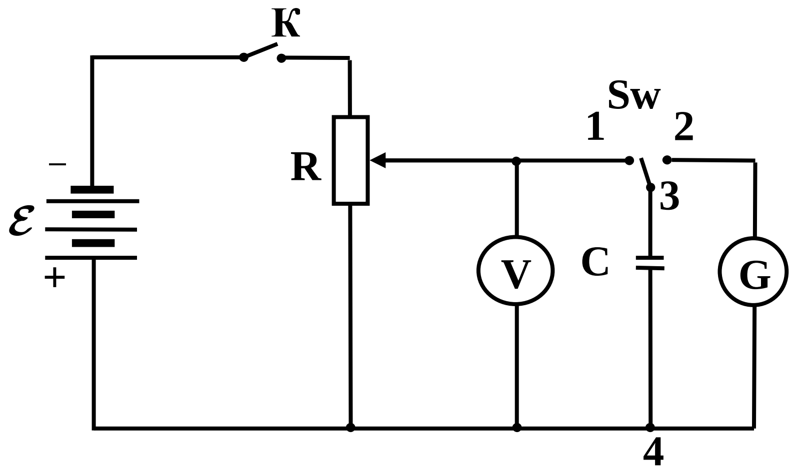


Figure 2.3.i

The circuit for the experiment is shown in Fig. 2.3.i. DC source $\mathcal{E}$ is connected to the potentiometer R. Potential difference U across the capacitor C is regulated by the potentiometer and measured by the voltmeter **V**. With the switch **Sw**, one breaks the circuit for capacitor charging and connects the capacitor into the circuit with the galvanometer **G** through which the capacitor is being discharged. By using the capacitor with known capacitance C_0, we can determine the galvanometer constant B. It is easily derived from Eqs. (2.3.3) and (2.3.1):

$$B = \frac{q_0}{n_0} = \frac{C_0 U}{n_0}, \tag{2.3.4}$$

Table 2.3.i

$U = ...$ V	Measurement #	1	2	3	4	5	$\langle n_0 \rangle$, g.p.
	n_0, g.p.						

where n_0 stands for galvanometer readings (in scale divisions) when the capacitor C_0 is being discharged.

Replacing the capacitor C_0 with the capacitor (or battery of capacitors) with unknown capacitance C_x, the latter can be determined by the formula obtained from the Eqs. (2.3.2), (2.3.3) and (2.3.4):

$$C_x = \frac{Bn_x}{U} = \frac{C_0 U}{n_0}\frac{n_x}{U} = C_0\frac{n_x}{n_0}, \tag{2.3.5}$$

where n_x stands for galvanometer readings under discharging of the capacitor (or battery of capacitors) C_x.

Step-by-step Procedure of the Experiment

1. Turn the handle of the potentiometer R to the leftmost position. Connect the capacitor with known capacitance C_0 to points 3, 4 of the electrical circuit with leads. Set the switch **Sw** at the position 1.
2. Switch on the power supply. Using the potentiometer set potential difference across the capacitor for its charging (the value U is recorded on the facility).
3. Measure the first throw position of the galvanometer light spot n_0 by turning the switch **Sw** from the position 1 (charging) to the position 2 (discharging). Repeat five times at the same potential difference U. (Charge the capacitor during 30 sec.) Calculate an average value of galvanometer readings $\langle n_0 \rangle$. Record all data to Table 2.3.i.
4. Connect the capacitor with unknown capacity C_x to the points 3, 4 of the circuit and repeat steps 2 and 3. Calculate an

Table 2.3.ii

$U = \ldots$ V	Measurement #	1	2	3	4	5	$\langle n_x \rangle$, g.p.
	n_x, g.p.						

average value of galvanometer readings $\langle n_x \rangle$. Record all data to Table 2.3.ii.

5. Calculate the capacitor capacitance C_x using Eq. (2.3.5).
6. Connect in parallel the capacitors C_0 and C_x, repeat steps 4, 5, and determine the capacitance of the capacitor battery C_{par}^{exp}.
7. Calculate the theoretical value of the capacitance of the capacitor battery using the formula

$$C_{par} = \sum_{i=1}^{N} C_i = C_0 + C_x,$$

and compare it with C_{par}^{exp}.

8. Connect in series the capacitors C_0 and C_x, repeat steps 4, 5, and determine the capacitance of this capacitor battery C_{ser}^{exp}.
9. Calculate the theoretical value of the capacitance of the capacitor battery using the formula

$$C_{ser} = \left(\sum_{i=1}^{N} \frac{1}{C_i} \right)^{-1} = \frac{C_0 C_x}{C_0 + C_x},$$

and compare it with C_{ser}^{exp}.

10. Analyze obtained results and make conclusions.

After-lab Questions

Case 1.

1. What is the capacitance of an isolated conductor, a capacitor?
2. Derive an expression for calculating the capacitance of capacitors connected in series.

3. What amount of energy is stored in a charged capacitor?
4. *Problem.* Calculate the capacity of a spherical capacitor which consists of two concentric spheres with radii $R_1 = 10$ cm and $R_2 = 10.5$ cm. The space between the spheres is filled up with oil of permittivity $\varepsilon = 2$. What radius should a single metal ball with the same capacitance have if it is placed in oil?

   Answer: $C = 470$ pF, $R = 2.1$ m.

Case 2.

1. We say that a capacitor has a charge Q. But is that what we really mean? Is the total charge on a capacitor actually Q?
2. Derive an expression for calculating the capacitance of capacitors connected in parallel.
3. What happens if you short out (connect with a conductor) the two plates of a large charged capacitor? Could this be dangerous?
4. *Problem.* The spheres with radii $R_1 = 30$ cm and $R_2 = 6$ cm have the same charges: $q_1 = q_2 = 2 \cdot 10^{-8}$ C. What are the potentials of the spheres? What potential will the spheres have if connected with a lead with negligible capacitance?
   Answer: $\varphi_1 = 600$ V, $\varphi_2 = 3000$ V, $\varphi = 1000$ V.

Case 3.

1. How does a dielectric placed in a capacitor change its capacitance? Why?
2. What is the expression for calculating the energy density for an electric field?
3. Derive an expression for calculating the capacitance of a parallel-plate capacitor.
4. *Problem.* A capacitor of capacitance $C_1 = 1 \cdot 10^{-9}$ F is charged to the potential difference of $U = 100$ V and connected in parallel with a discharged capacitor of capacitance $C_2 = 2 \cdot 10^{-9}$ F.

How much energy ΔW is lost in spark generation because of the connection?

Answer: $\Delta W = 3.3\ \mu$J.

LABORATORY EXPERIMENT 2.4

INVESTIGATION OF THE PROCESSES OF CAPACITOR'S CHARGING AND DISCHARGING

Purpose of the Experiment: to investigate in experimental the processes of capacitor charging and discharging and determine the time constant.

Equipment and Accessories: electronic oscilloscope with rectangular pulse generator; sets of capacitors and resistances.

Basic Methodology. A capacitor is charged and discharged through a resistor. At a special mode of operation of the oscilloscope, at its screen we can see the dependence of the potential difference across the capacitor on time.

Recommended Pre-lab Reading: [1] 26.4; [2] 27.5; [3] 28.4.

Pre-lab Questions

1. What is the relationship between the potential difference across the capacitor plates and the charge stored on its plates?
2. What is the relationship between the current going through the resistor and the voltage across it?
3. Use these relationships to write Kirchhoff loop equation in terms of the EMF of the source, the capacitance of the capacitor, the resistance of the resistor, the current going through the circuit, and the charge stored on the capacitor.

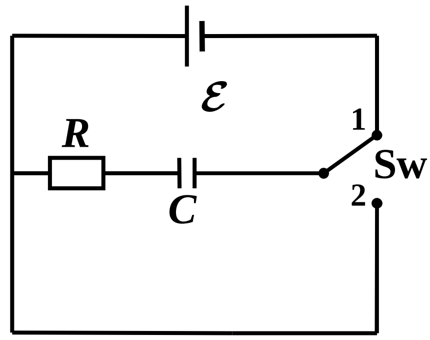


Figure 2.4.i

Theoretical Introduction

The basic circuit for examining processes of the capacitor charging and discharging is shown in Fig. 2.4.i. It consists of a DC source with the electro-motive force $\mathcal{E}$, a capacitor C, and a resistance R in series. By using a switch **Sw**, the capacitor can be connected to the source (position 1, the capacitor is being charged) or disconnected from it (position 2, the capacitor is being discharged).

Let us consider the process of the capacitor's charging. When the circuit with the source and the capacitor is closed (**Sw** is in position 1), the capacitor is being charged and there is a time-varying current in the circuit. According to Kirchoff's second rule (Kirchoff's loop rule), the sum of the potential difference across the capacitor C and the voltage across the resistor R is equal to the EMF of the source $\mathcal{E}$:

$$\mathcal{E} = V_C + V_R = \frac{q}{C} + IR, \tag{2.4.1}$$

where q is the capacitor's charge; I is the current at the instant. While the capacitor is being charged, its charge q is increasing and the current is decreasing. We can evaluate the charge dependence on time using the formula (2.4.1). As $I = \dfrac{dq}{dt}$, so

$$\mathcal{E} = \frac{q}{C} + R\frac{dq}{dt}. \tag{2.4.2}$$

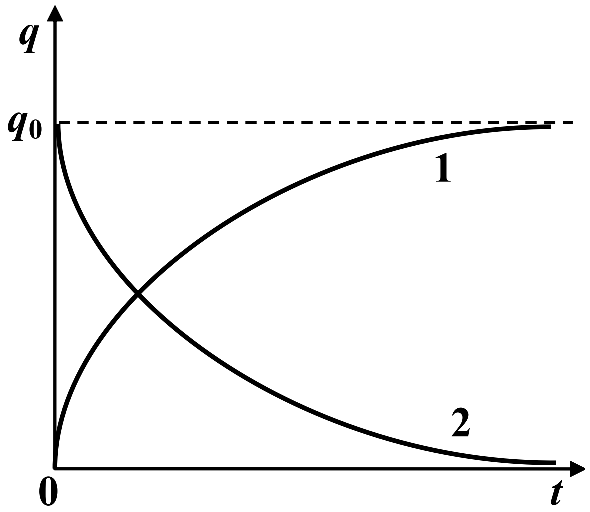


Figure 2.4.ii

Taking into account the initial condition $q(t)\big|_{t=0} = 0$, the solution of this differential equation is the function

$$q(t) = q_0 \left(1 - \exp^{-t/RC}\right), \tag{2.4.3}$$

where $q_0 = \mathcal{E}C$ is the maximum charge of the capacitor.

Eq. (2.4.3) is the time dependence of the capacitor charge on the charging process. The graph of this dependence is shown in Fig. 2.4.ii (curve 1).

For capacitor discharging, the switch is in the position 2 (see Fig. 2.4.i). In this case, there is no source in the closed circuit and Eq. (2.4.1) looks like

$$\frac{q}{C} + IR = 0,$$

or (as $I = \dfrac{dq}{dt}$)

$$\frac{dq}{dt} + \frac{q}{RC} = 0.$$

Taking into account the initial condition $q(t)\big|_{t=0} = q_0$, the solution of this equation is the function

$$q(t) = q_0 \exp^{-t/RC}. \tag{2.4.4}$$

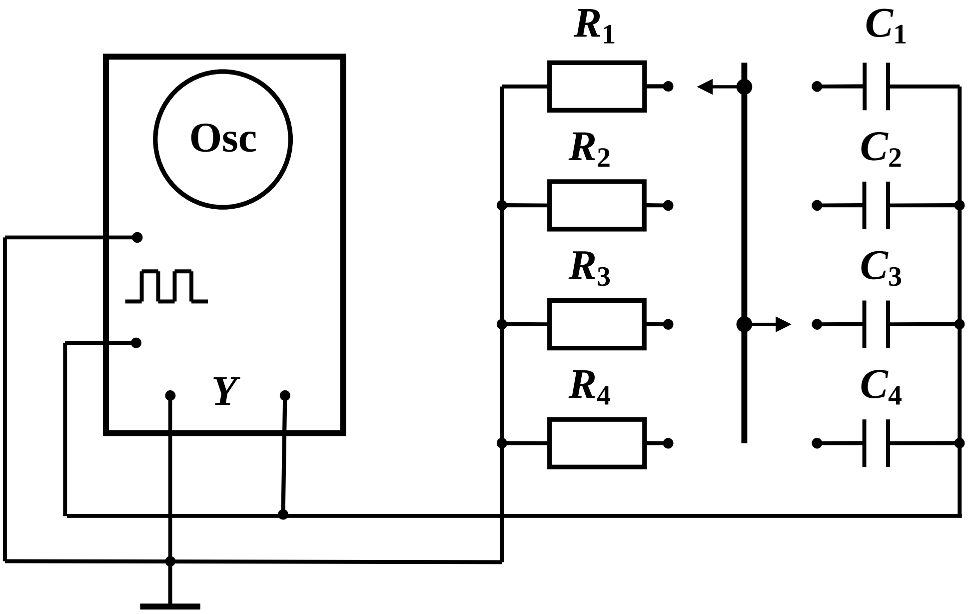


Figure 2.4.iii

Eq. (2.4.4) is the time dependence of capacitor charge on the discharging process. The graph of this dependence is shown in Fig. 2.4.ii (curve 2).

The rate of the capacitor charging (discharging) is determined by the time constant (or the relaxation time) of RC–circuit: $\tau = RC$. τ is the time interval after disconnection with EMF during which the capacitor charge has decreased to $\frac{1}{e}$ (about 0.368) of its initial value.

The scheme of laboratory setup is shown in Fig. 2.4.iii. Connection and disconnection to the source is imitated by the rectangular pulsing from generator to the RC–circuit. Rectangular pulse generator is included into the oscilloscope. During the time t_{inc}, rectangular pulse generator supplies the voltage U_0 constant in magnitude(Fig. 2.4.iv, curve 1). This corresponds to the circuit for capacitor charging and, therefore, increase of the voltage across the capacitor. Then, during the time t_{dec}, the generator supplies a zero voltage. This corresponds to the circuit for capacitor discharging and, therefore, decrease of the voltage across the capacitor. Processes of charging and discharging repeat themselves

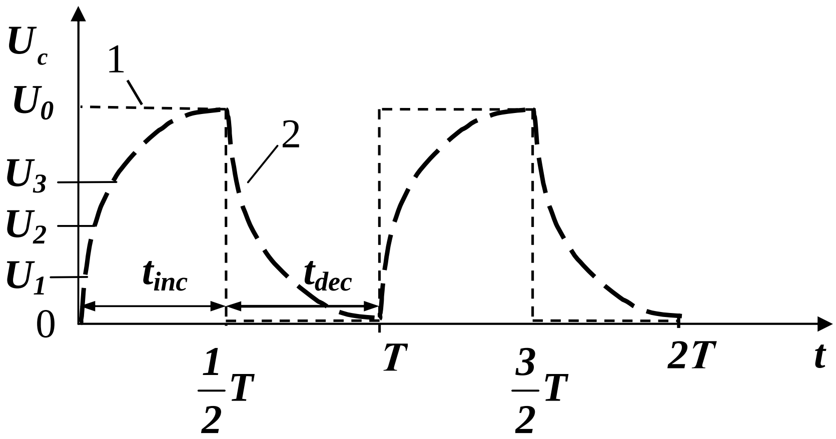


Figure 2.4.iv

with the period $T = t_{inc} + t_{dec}$ (see Fig. 2.4.iv). In this experiment, the generator is adjusted so that the time of capacitor charging is equal to the time of capacitor discharging: $t_{inc} = t_{dec} = \dfrac{T}{2}$. Connecting a resistor R_i and a capacitor C_j in series to the generator, one can examine the processes of capacitor charging and discharging at different values of $\tau = R_i C_j$.

The voltage across the capacitor is applied to the oscilloscope input and, for the given RC–circuit, the time dependence of voltage $U_C(t)$ is shown at the oscilloscope screen. As the voltage across the capacitor plates is proportional to its charge $U_C(t) = \dfrac{q(t)}{C}$, the dependence $U_C(t)$ is similar to the dependence of the capacitor charge on time $q(t)$ (see Fig. 2.4.iv, curve 2).

For quantitative investigation of this dependence we can use the oscilloscope mode of operation when a continuous curve is shown as a dashed line (the mode "Marks", in Russian "Метки") (see Fig. 2.4.iv).

The time interval between the marks Δt_m is determined by the position of the switch "Метки" ($\Delta t_m = 20$ or $100\ \mu s$), and it is a distance between the beginnings of the adjacent marks (segments of line) along the horizontal axis of time. By using marks, we can determine the time of voltage increase (decrease) across the capacitor under its charging (discharging). For example, if within the period $0 - t_{inc}$ there are six full marks, and the time interval between the marks $\Delta t_m = 20\ \mu s$, then the time of voltage increase under the capacitor charging is $t_{inc} = 20 \cdot 6 = 120\ \mu$s. In Fig. 2.4.iv ordinates of the curve points U_k which correspond to the instants of time $t_k = k\Delta t_m$ ($k = 1,\ 2, \ldots$), give the dependence of voltage across the capacitor on time

$$U_k = f(t). \tag{2.4.5}$$

These data are used for time constant τ computing.

Taking the logarithm of the Eqs. (2.4.3) and (2.4.4), we obtain

$$\ln \frac{q_0}{q_0 - q} = \frac{t}{RC} = \frac{t}{\tau}, \quad \ln \frac{q_0}{q} = \frac{t}{\tau}.$$

As the charge q is proportional to the voltage across the capacitor U_C, so we can write:

$$\ln \frac{U_0}{U_0 - U_k} = \frac{t}{\tau} \quad \text{(for charging);} \tag{2.4.6}$$

$$\ln \frac{U_0}{U_k} = \frac{t}{\tau} \quad \text{(for discharging).} \tag{2.4.7}$$

To calculate the time constant τ, you need to construct graphs of the dependence of $\ln \dfrac{U_0}{U_0 - U_k}$ (for charging) and $\ln \dfrac{U_0}{U_k}$ (for discharging) on time t. The value of τ is determined by using an appropriate graph. For example, for capacitor charging (see Fig. 2.4.v)

$$\tau_{exp} = \frac{\Delta t}{\Delta \left(\ln \dfrac{U_0}{U_0 - U_k} \right)}. \tag{2.4.8}$$

By analogy, for capacitor discharging

$$\tau_{exp} = \frac{\Delta t}{\Delta \left(\ln \dfrac{U_0}{U_k} \right)}. \tag{2.4.9}$$

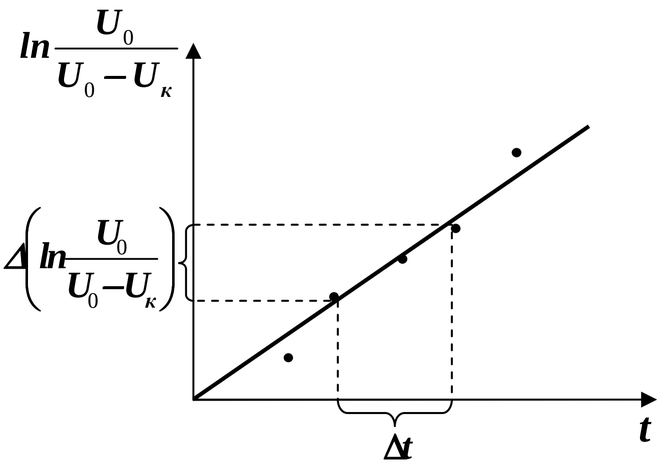


Figure 2.4.v

Δt is an arbitrarily chosen time interval, and the change of functions $\Delta\left(\ln\dfrac{U_0}{U_0-U_k}\right)$ (for charging) or $\Delta\left(\ln\dfrac{U_0}{U_k}\right)$ (for discharging) occurs within this time interval.

Step-by-step Procedure of the Experiment

Task 1. Determination of the time constant for the process of capacitor charging.

1. Connect a resistor R_i and a capacitor C_j to the circuit (the values of R_i and C_j are given by the instructor).
2. Switch on the setup and the oscilloscope (it should warm up for 2–3 minutes). Get a stable picture of the dependence of voltage across the capacitor on time $U_C(t)$ (see Fig. 2.4.iv).
3. Determine time coordinates of the marks on the curve of voltage increase by the formula $t_k = k\Delta t_m$, where k is the number of a mark, Δt_m is the time interval between the adjacent marks (is given by the instructor).
4. Determine the values of the ordinates U_k of the marks' beginnings on the curve $U_C(t)$ and the maximum value of voltage U_0 (see Fig. 2.4.iv). Write the data into Table 2.4.i.
5. Construct a graph of the dependence $\ln\dfrac{U_0}{U_0-U_k}$ on time t. Using it and Eq. (2.4.8), compute the value of τ_{exp}.

6. Calculate the theoretical value of the time constant according to the formula $\tau_{th} = R_i C_j$ and compare it with τ_{exp}.
7. Make conclusions.

Table 2.4.i

Label #, k	$t_k = k\Delta t_m$, μs	U_k, g.p.	U_0, g.p.	$\ln \frac{U_0}{U_0 - U_k}$	τ_{exp}, μs	τ_{th}, μs
0	0	0		0		
1						
2						
3						
4						
5						

Task 2. Determination of the time constant for the process of capacitor discharging.

1. Connect a resistor R_i and a capacitor C_j to the circuit (the values of R_i and C_j are given by the instructor).
2. Switch on the setup and the oscilloscope (it should warm up for 2–3 minutes). Get a stable picture of the dependence of voltage across the capacitor on time $U_C(t)$ (see Fig. 2.4.iv).
3. Determine time coordinates of the marks on the curve of voltage decrease by the formula $t_k = k\Delta t_m$, where k is the number of a mark, Δt_m is the time interval between the adjacent marks (is given by the instructor).
4. Determine the values of the ordinates U_k of the marks beginnings on the curve $U_C(t)$ and the maximum value of voltage U_0. Write the data into Table 2.4.ii.
5. Construct a graph of the dependence $\ln \dfrac{U_0}{U_k}$ on time t. Using it and Eq. (2.4.9), compute the value of τ_{exp}.

6. Calculate the theoretical value of the time constant according to the formula $\tau_{th} = R_i C_j$ and compare it with τ_{exp}.
7. Make conclusions.

Table 2.4.ii

Label #, k	$t_k = k\Delta t_m$, μs	U_k, g.p.	U_0, g.p.	$\ln \frac{U_0}{U_k}$	τ_{exp}, μs	τ_{th}, μs
0	0			0		
1						
2						
3						
4						
5						

After-lab Questions

Case 1.

1. What is a capacitor? Give the definition of the capacitor capacitance.
2. What is the time constant of RC–circuit? What is its physical meaning?
3. Prove that the time constant τ of RC–circuit has the dimension of time.
4. *Problem.* A capacitor with the capacitance $C = 200$ μF and a resistor with the resistance $R = 12$ Ω are connected in series to the DC source. Find the time during which the voltage across the capacitor has decreased to a half of its initial value.

   Answer: $t = 1.67$ ms.

Case 2.

1. What is a parallel-plate capacitor? Derive a formula for the capacitance of a parallel-plate capacitor?

2. Write the equations for the dependence of the capacitor charge on time during the capacitor charging and discharging.
3. A parallel-plate air capacitor is charged and then disconnected from the source of charge. What is the change in the energy stored in the capacitor if a slab of dielectric (with the dielectric constant $K > 1$) is inserted between its plates, completely filling this space?
4. *Problem.* A capacitor with the capacitance $C = 3\ \mu$F and a resistor with the resistance $R = 10\ \Omega$ are connected in series to the DC source with $\mathcal{E} = 2$ V. How much charge has been accumulated on the capacitor during the time $t = 1.5\ \mu$s?
Answer: $q = 0.29\ \mu$C.

Case 3.

1. What is the capacitance of an isolated conductor? Derive a formula for calculation the capacitance of an isolated sphere.
2. Write the differential equations and their solutions for the processes of the capacitor charging and discharging.
3. What parameters of the circuit influence the processes of the capacitor charging and discharging?
4. *Problem.* A capacitor with the capacitance 100 μF is being discharged through a resistor. What is the resistance of the resistor, if within 2 ms the charge on the capacitor has decreased to a half of its initial value.
Answer: $R = 28.9\ \Omega$.

LABORATORY EXPERIMENT 2.5

DETERMINATION OF EMF OF THE DC SOURCE BY THE COMPENSATION METHOD

Purpose of the Experiment: to get acquainted with the compensation method and its application for EMF measuring of the DC source.

Equipment and Accessories: Veston's standard cell; cell with an unknown EMF (a battery); reference-voltage source; two resistance boxes; microammeter.

Basic Methodology. There is a branching circuit with a voltage reference, resistance boxes, and two EMF (known and unknown) which are in parallel and connected to the circuit in turn. The unknown EMF can be determined if we (1) connect the known EMF and adjust the resistances so that there is no current through the specified subcircuit; (2) repeat the same steps with the unknown EMF.

Recommended Pre-lab Reading: [1] 25.4, 26.2; [2] 27.1–27.3; [3] 28.1, 28.3.

Pre-lab Questions

1. What is the SI unit for EMF?
2. What is the role of the source of EMF in a complete circuit?
3. What kind of forces exist inside the source of EMF? Can it be an electrostatic force?

Theoretical Introduction

Compensation methods are widely used for measuring electrical quantities due to their universality (you can measure EMF, voltage, current, resistance, and power), reliability, and high accuracy of measurements. The main idea of the method is to balance the measured EMF by the voltage drop across the certain subcircuit in such a way that there is no current through the subcircuit with the EMF.

A simplified diagram of the circuit intended for EMF measurement using the compensation method is shown in Fig. 2.5.i. Current moves from the reference-voltage source $\mathcal{E}_0$ through the resistance R which consists of two variable resistances R_1 and R_2

($R = R_1 + R_2$). A cell with known EMF, $\mathcal{E}_N$, is connected to the node B through the microammeter (the source $\mathcal{E}_N$ and the cell $\mathcal{E}_0$ are connected to the node A by the analogous poles). Resistances of the elements AB (R_1) and BC (R_2) can be adjusted so that the voltage drop across the element AB is equal to the EMF of the cell $\mathcal{E}_N$ (meanwhile their total resistance R is unchangeable). In this case, the microammeter shows no current.

According to the first Kirchhoff's rule for the node A,

$$I_1 + I_2 - I = 0. \tag{2.5.1}$$

By using the second Kirchhoff's rule for the loop B$\mathcal{E}_N$AB, we get the following equation:

$$\mathcal{E}_N = I_2(r + R_A) + IR_1, \tag{2.5.2}$$

where r is the internal resistance of the cell $\mathcal{E}_N$; R_A is the resistance of the microammeter.

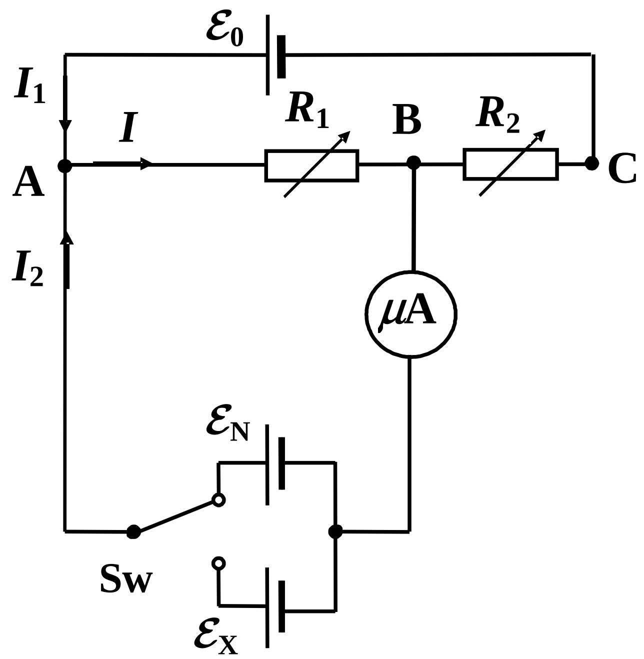


Figure 2.5.i

If the compensation has been achieved, there is no current in the above mentioned loop ($I_2 = 0$) and Eqs. (2.5.1) and (2.5.2)

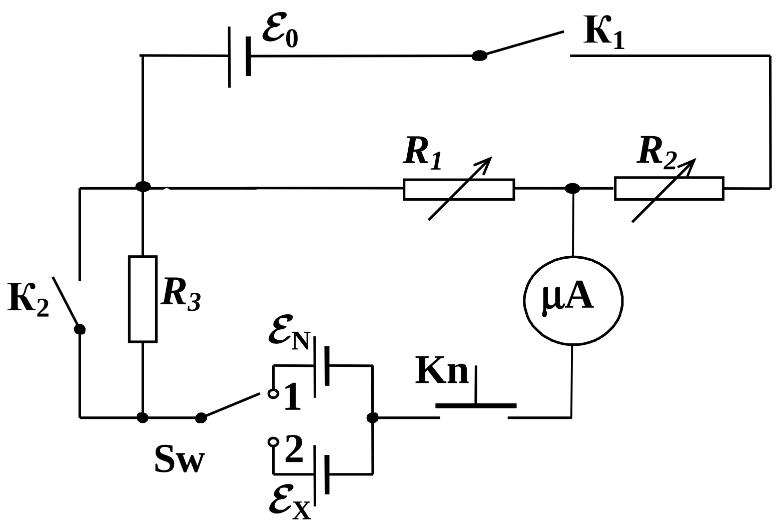


Figure 2.5.ii

have the following form:

$$I_1 = I; \quad \mathcal{E}_N = IR_1. \tag{2.5.3}$$

And now let's replace the cell $\mathcal{E}_N$ by using the switcher **Sw** (see Fig. 2.5.i) with the cell $\mathcal{E}_X$ with the unknown EMF and change the resistances R_1 and R_2 (the total R is unchangeable) so that no current will go through the microammeter. It holds under the new value of resistance of the element AB, and instead of Eq. (2.5.3) we can write

$$I_1 = I; \quad \mathcal{E}_X = IR_X. \tag{2.5.4}$$

From the Eqs. (2.5.3) and (2.5.4) we get a formula for an unknown EMF:

$$\mathcal{E}_X = \mathcal{E}_N \frac{R_X}{R_1}. \tag{2.5.5}$$

The circuit diagram of the laboratory installation is shown in the Fig. 2.5.ii. The standard Veston's cell is used as the known EMF $\mathcal{E}_N$. It is a variant of galvanic cells. The potential difference between the electrodes of the Veston's cell arises because of the oxidation-reduction reactions which occur in the saturated solution of the cadmium sulphate with the addition of mercury and mercury sulphate. The electromotive force of the Veston's cell is reproduced with fine precision that is why it is used as a standard. At 20°C, its EMF is $\mathcal{E}_N = 1.0183$ V. A battery is used as a cell with unknown EMF $\mathcal{E}_X$. It is connected to the reference-voltage source $\mathcal{E}_0$ by the same poles as the standard cell $\mathcal{E}_N$. The key $\mathbf{K}_1$ switches the reference-voltage source. The knob **Kn** is for short-term closing of circuit of the microammeter. R_3 is a high (about 10^4 Ω) resistance which protects the microammeter and the standard cell from high currents going through them under a preliminary rough compensation. The key $\mathbf{K}_2$ is used for short-circuit of the resistance R_3 under an accurate compensation. Resistance boxes R_1 and R_2 are used as variable resistances. The compensation is achieved by the adjustment of resistances under the condition of a constancy of their total resistance.

Step-by-step Procedure of the Experiment

1. Set the value of 20 kΩ to each of resistance boxes R_1 and R_2.
2. Place the battery into a jack for unknown EMF (REMEMBER: its polarity in the circuit should be the same as the polarity of the cell $\mathcal{E}_N$).
3. Close the key $\mathbf{K}_1$.
4. Set the switcher **Sw** in the position 1 for standard cell connecting to the circuit. The key $\mathbf{K}_2$ is opened (that is the position "Rough" ("Грубо")).
5. Obtain no current through the microammeter during the short-term pressing of the button **Kn**. For this, use three first decades of the resistance boxes (×10000, ×1000, ×100) to change the resistances R_1 and R_2 so that their sum remains constant, i.e. 40 kΩ.
6. Obtain more accurate compensation of the current through the microammeter, by closing the key $\mathbf{K}_2$ (the position "Accurate" ("Точно")) and repeating step 5. Use other decades of

resistance boxes. Record the value R_1 of the first resistance box when there is no current through the microammeter.

7. With the switcher **Sw**, switch the battery in the circuit instead of the standard cell $\mathcal{E}_N$.
8. Repeat steps 5 and 6, and find out the value of the resistance R_X of the first resistance box in this case.
9. Using Eq. (2.5.5) calculate the EMF of a battery.
10. Make conclusions.

After-lab Questions

Case 1.

1. What conditions are necessary for a steady electrical current to exist?
2. Write down the Ohm's law for a complete circuit.
3. Explain why the direct measurement of EMF of the source with voltmeter is not accurate.
4. *Problem.* Firstly a source of current was connected to the external resistance $R_1 = 2\ \Omega$, and then, to the external resistance $R_2 = 0.5\ \Omega$. In each case power P which is extracted in the external circuit is the same and equal to 2 W. Find the EMF of a source and its internal resistance.
   Answer: $\mathcal{E} = 3$ V, $r = 1\ \Omega$.

Case 2.

1. What is an electromotive force of a source and voltage across a circuit element?
2. Formulate and write down Kirchhoff's rules.
3. Write down the Ohm's law for current density.
4. *Problem.* A copper transmission cable 100 km long and 10.0 cm in diameter carries a current of 125 A. (a) What is the potential drop across the cable? (b) How much electrical energy is dissipated as the thermal energy every hour? Resistivity of copper is $1.72 \cdot 10^{-8}\ \Omega$ m.
   Answer: a) $V = 27.39$ V, b) $E = 1.23 \cdot 10^7$ J.

Case 3.

1. Write down the Ohm's law for elements of a circuit with a source of EMF and without it.
2. What is a terminal voltage?
3. Derive an expression for power extracted from a circuit element.
4. *Problem.* Voltmeter and resistance are connected in series. The voltage across this circuit element is $U_0 = 120$ V. When the resistance value is $R = 10^4$ Ω, the voltmeter reading is $U_1 = 40$ V. What resistance R_x would be if the voltmeter reading is $U_2 = 10$ V.
   Answer: $R_x = 5.5 \cdot 10^4$ Ω.

LABORATORY EXPERIMENT 2.10

ELECTRON SPECIFIC CHARGE MEASUREMENTS BY MEANS MAGNETRON METHOD

Purpose of the Experiment: to determine electron specific charge and compare it with tabulated value.

Equipment and Accessories: vacuum diode; filament power source; adjustable power source of the anode circuit; solenoid and its power source; ammeters and voltmeter.

Basic Methodology. The critical value of the magnetic field that influences electron's motion in the vacuum diode is determined based on the shape of the anode current curve. The specific charge of the electron is calculated from the relations of the voltage applied to the diode, the magnitude of the critical magnetic field, and the radii of the electrodes of the diode.

Recommended Pre-lab Reading: [1] 27.4, 27.5; [2] 28.2, 28.3; [3] 29.2, 29.3.

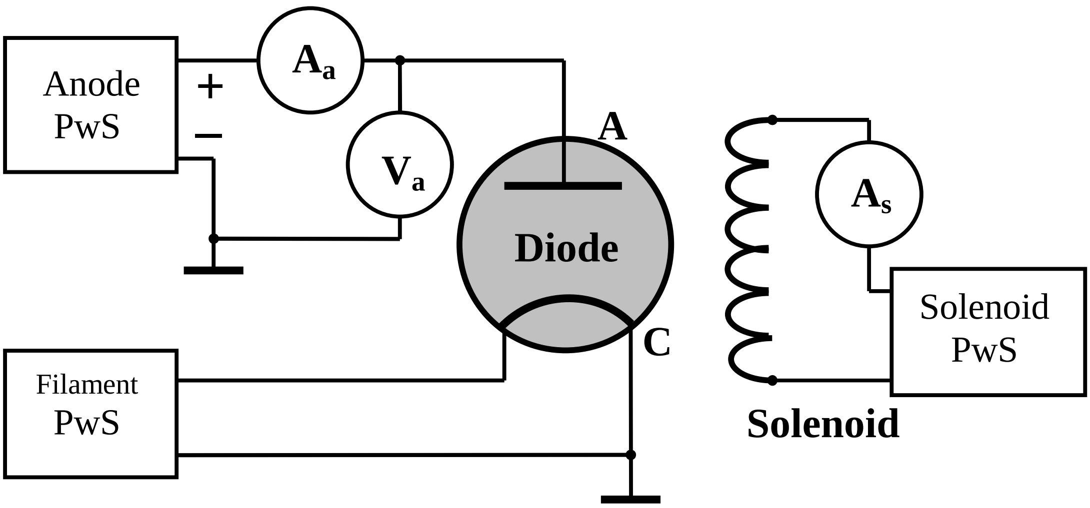


Figure 2.10.i

Pre-lab Questions

1. What is a specific charge of a particle?
2. What force acts upon the charge particle moving in the electric and magnetic field?
3. What is the magnetron?

Theoretical Introduction

Specific charge of an elementary particle is the ratio of the particle charge to its mass. To determine the electron specific charge, we will use the magnetron method. The method utilizes the features of charged particles motion in crossed electric and magnetic fields. Magnetron is a cylindrically symmetric vacuum diode placed inside a concentric solenoidal coil. The experiment is conducted with the setup shown in Fig. 2.10.i.

The main element of the installation is a vacuum diode with coaxial cylindrical electrodes: the cathode **C** and the anode **A** (Fig. 2.10.ii). The electrodes are placed in a glass tube. During its manufacturing, the tube was evacuated and sealed.

The cathode of the diode is heated by the direct current of a filament power source. The thermionic emission of electrons takes place from the surface of the heated cathode. Space charge region or electron cloud is created near the surface of the cathode.

The size of this region depends on the potential difference between cathode and anode. Emitted electrons in the space charge move randomly, and the velocity distribution of emitted electrons is Maxwellian. Electrons are pulled out of the space charge cloud by an external radial electric field (the field lines are shown in Fig. 2.10.ii). As a result, electrons are accelerated from the cathode toward the anode.

The magnitude of the electric field in the interelectrode space is given by an equation

$$E = -\frac{U_a}{r \ln \frac{r_a}{r_c}},$$

where U_a is the potential difference between anode and cathode (anode voltage); r_a and r_c are the radii of anode and cathode, respectively; $r$is the distance from the axis of the system to the considered point.

The vacuum diode is placed inside the solenoid. Uniform magnetic field is created by the current in the solenoid. The value of the magnetic field B can be adjusted by changing the solenoid current I_s. Vector $\vec{\boldsymbol{B}}$ is parallel to the diode axis of symmetry.

The Lorentz force $\vec{\boldsymbol{F}}_L$ is acting upon the charged particle with the charge $q = -e$ (e is the elementary charge) moving with the velocity $\vec{\boldsymbol{v}}$ in the the electric field $\vec{\boldsymbol{E}}$ and magnetic field $\vec{\boldsymbol{B}}$

$$\vec{\boldsymbol{F}}_L = -e\left(\vec{\boldsymbol{E}} + \vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}\right).$$

According to the Newton's second law,

$$m\frac{d\vec{\boldsymbol{v}}}{dt} = \vec{\boldsymbol{F}}_L,$$

where m is the mass of a particle. Thus, the equation of the electron motion is

$$\frac{d\vec{\boldsymbol{v}}}{dt} = -\frac{e}{m}\left(\vec{\boldsymbol{E}} + \vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}\right). \tag{2.10.1}$$

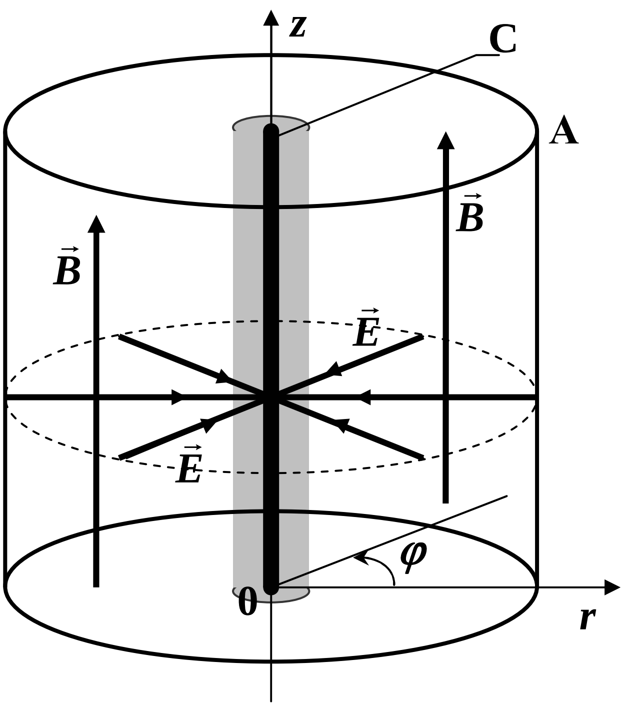


Figure 2.10.ii

To describe the motion of an electron in the gap between coaxial cylindrical electrodes, it is convenient to use a cylindrical coordinate system (r, φ, z) with Z-axis aligned along the uniform magnetic field $\vec{\boldsymbol{B}}$ (see Fig. 2.10.ii). The projections of Eq. (2.10.1) in this coordinate system is the following

$$\begin{aligned} \frac{d^2 z}{dt^2} &= 0, \\ \frac{d^2 r}{dt^2} - r\left(\frac{d\varphi}{dt}\right)^2 &= \frac{e}{m}\left(E - r\frac{d\varphi}{dt}B\right), \\ 2\frac{dr}{dt}\frac{d\varphi}{dt} + r\frac{d^2\varphi}{dt^2} &= \frac{e}{m}\frac{dr}{dt}B, \end{aligned} \tag{2.10.2}$$

where $\dfrac{dz}{dt} = v_z$, $\dfrac{dr}{dt} = v_r$, and $r\dfrac{d\varphi}{dt} = v_\varphi$ is an azimuthal, radial, and polar component of the velocity vector $\vec{\boldsymbol{v}}$ respectively.

Lets assume that the initial velocity of an electron at the cathode surface is zero. Then an electron will move only in the plane (r, φ) under the influence of the Lorentz force. Solving the last equation of the set (2.10.2), we obtain

$$\frac{d\varphi}{dt} = \frac{eB}{2m}\left(1 - \frac{r_c^2}{r^2}\right). \tag{2.10.3}$$

Since the magnetic component of the Lorentz force does not perform work on charged particles, the electron energy is gained only from the electric field. The work eU_a done by the electric field on an electron when moving from cathode to anode is equal to electron kinetic energy

$$eU_a = \frac{m}{2}\left(\upsilon_r^2 + \upsilon_\varphi^2\right). \tag{2.10.4}$$

The Pythagorean theorem is applied to the speed of electron, since radial and polar components of the electron velocity are mutually perpendicular. Substituting (2.10.3) into Eq. (2.10.4) we obtain:

$$eU_a = \frac{m}{2}\left(\upsilon_r^2 + \frac{e^2B^2}{4m^2r_a^2}\left(r_a^2 - r_c^2\right)^2\right). \tag{2.10.5}$$

Lets consider the trajectory of an electron in the inter-electrode gap with the potential difference U_a (Fig. 2.10.iii).

Electrons move radially from cathode to anode if no magnetic field is applied (trajectory 1). The trajectory of electrons is curved (trajectory 2) for a non-zero magnetic field. If the magnetic field becomes strong enough, the electrons can never reach the anode (trajectory 3′). The trajectory of an electron is tangent to the anode (trajectories 3) at a certain critical value of magnetic field B_{cr}. Obviously, when $B = B_{cr}$, the radial component of the electron velocity vanishes and Eq. (2.10.5) takes the form

$$eU_a = \frac{e^2B_{cr}^2}{8mr_a^2}\left(r_a^2 - r_c^2\right)^2. \tag{2.10.6}$$

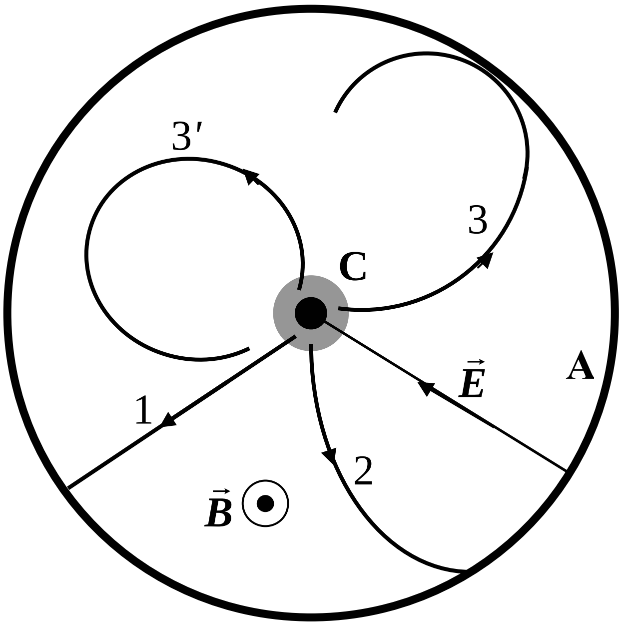


Figure 2.10.iii

Thus, to determine the specific charge of an electron, one can use Eq. (2.10.6) if the value of B_{cr} is measured:

$$\frac{e}{m} = \frac{8U_a r_a^2}{B_{cr}^2 \left(r_a^2 - r_c^2\right)^2}. \tag{2.10.7}$$

There is a current I_a in a vacuum diode due to the motion of thermal electrons from cathode to anode. If the magnetic field B is stronger than the critical value B_{cr}, the electrons form a space charge region around the cathode, screening it, and the current going through the diode drops to zero. Fig. 2.10.iv shows the dependence of the diode current I_a on the magnitude of the magnetic filed B of the solenoid. The dashed curve corresponds to the case when the initial velocity of electrons is zero at the cathode surface, as previously assumed. The critical conditions are reached for different electrons under different values of magnetic field in a real case of a Maxwellian distribution of the initial electron velocities. In this case the dependence of $I_a(B)$ is the curve shown in Fig. 2.10.iv solid line.

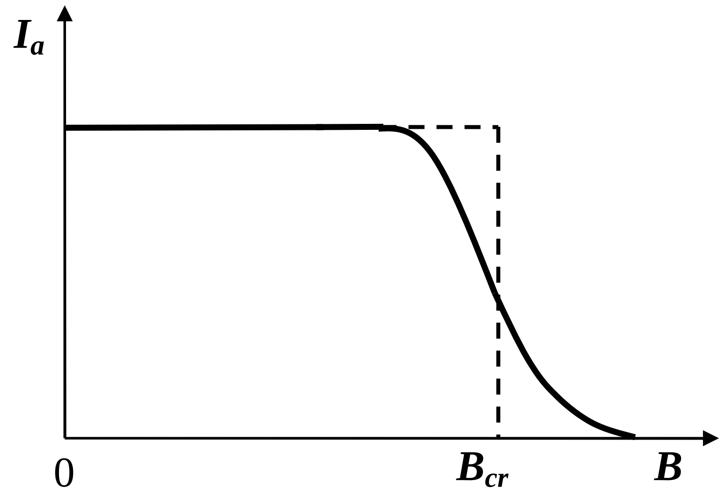


Figure 2.10.iv

Not ideal cylindrical symmetry of the diode, alignment of lamp electrodes and solenoid, and a number of other factors lead to an additional smoothing of the curve. Nevertheless, the curve $I_a(B)$ bend is sufficiently sharp to determine the critical value of the magnetic field.

Step-by-step Procedure of the Experiment

1. Check that the elements are connected according to the scheme depicted in Fig. 2.10.i. The handles of voltage regulators of the diode and the solenoid must be turned counterclockwise until the end. Turn on the equipment.
2. Turn on the power source of anode voltage and establish the value of anode voltage U_a, indicated on the equipment. Wait five minutes until the cathode is warmed up. While conducting measurements, keep an eye on the anode voltage. It must be constant through the experiment.
3. Turn on the power supply of the solenoid. Varying the value of the current in the solenoid from zero to the maximum possible one, take the dependence of the anode current vs. the current in the solenoid $I_a(I_s)$.
4. Plot a graph I_a versus I_s, based on the data you have measured.

5. Determine the critical solenoid current $I_{s.cr}$ that corresponds to the highest slope of the curve $I_a(I_s)$.
6. Calculate the critical value of the magnetic filed B_{cr} based on the found value $I_{s.cr}$ with the help of the expression

$$B_{cr} = kI_{s.cr}.$$

   The value of constant k specified at the equipment.
7. Calculate the value of the electron specific charge according to Eq. (2.10.7).
8. Analyze the results, make conclusions.

After-lab Questions

Case 1.

1. Write an expression for the force exerted on a charged particle in the electric field.
2. Explain why the magnetic force does not perform work on a charge particle.
3. An electron enters a uniform magnetic field at a sharp angle to the field lines. Draw its trajectory. Explain it.
4. *Problem.* Proton and electron enter a uniform magnetic field perpendicular to the field lines. Speeds of the particles are the same. What is the ratio of the radii of curvature of the proton and the electron trajectories?
   Answer: $R_p/R_e = 1836$.

Case 2.

1. Write an expression for the force exerted on a charged particle moving in the magnetic field.
2. How does the work of the force that acts on a charged particle moving in an electric field defined?
3. Proton enters a uniform crossed electric and magnetic fields along the electric field lines. Draw the trajectory of its motion. Explain the behavior of the proton.

4. *Problem.* The cyclotron is designed to accelerate protons to the energies of 5 MeV. What is the smallest radius R of the cyclotron dees. The magnetic field $B = 1$ T.
   Answer: $R > 0.32$ m.

Case 3.
1. Write an expression for the Lorentz force.
2. α–particle enters uniform magnetic field along the field line. Explain the behavior of the particle and draw its trajectory.
3. Make examples of using the technical characteristics of the motion of charged particles in crossed electric and magnetic fields.
4. *Problem.* An electron enters a uniform magnetic field perpendicular to the field lines. Find the cyclotron frequency if the magnetic field $B = 2 \cdot 10^{-2}$ T.
   Answer: $\omega = 3.5 \cdot 10^9$ rad/s.

LABORATORY EXPERIMENT 2.13

MAGNETIC FIELD OF A COIL AND SYSTEMS OF TWO COILS

Purpose of the Experiment: to investigate the magnetic field at the axis of the short coil, to verify the Biot-Savart law and the superposition principle.

Equipment and Accessories: two identical short coils; AC current source; ammeter; probe for measuring magnetic field; millivoltmeter.

Basic Methodology. The value of EMF induced in a small probe-coil is proportional to the peak magnitude of magnetic filed created by a circular coil conducting an AC-current. By measuring this EMF for different positions of the probe, the time varying magnetic field produced by the large field coil can be mapped out.

Recommended Pre-lab Reading: [1] 28.2, 28.5, 29.2; [2] 29.4, 30.1; [3] 30.1; 31.1.

Pre-lab Questions

1. What unit is used to measure magnetic filed?
2. Write an expression for Biot-Savart law. Draw a picture to explain it.
3. Formulate Faraday's law.

Theoretical Introduction

The magnetic field of a coil and a system of two coils is investigated with a setup shown schematically in Fig. 2.13.i. Two coils L_1 and L_2 are coaxial and have the same parameters: the average radius $R = 6.5$ cm, the length of coils $l = 2.5$ cm, and the number of turns $N_C = 290$. The coil L_1 is a motionless solenoid while the position of L_2 is changeable along the X-axis of the system. The probe Z is used to measure the magnetic filed created by a coil. The position of the probe Z is also changeable along the axis of the coils. The separation between coils and the probe position is determined by the scale on their holders.

The magnetic field measurements are based on the the phenomenon of electromagnetic induction. The probe is a small coil with a large number of turns. If the probe is placed in an alternating magnetic field, according to Faraday's law, the EMF would be induced in it:

$$\mathcal{E}_i = -\frac{d\Psi_m}{dt} = -NA\frac{dB}{dt},$$

where Ψ is a net magnetic flux; A is a cross-section area of the probe coil; N is a the number of the turns in the probe. Since probe is small in-seize, we assume that the magnetic field B is uniform along the area of the probe. The cross-section of the search coil is normal to the axis of the system.

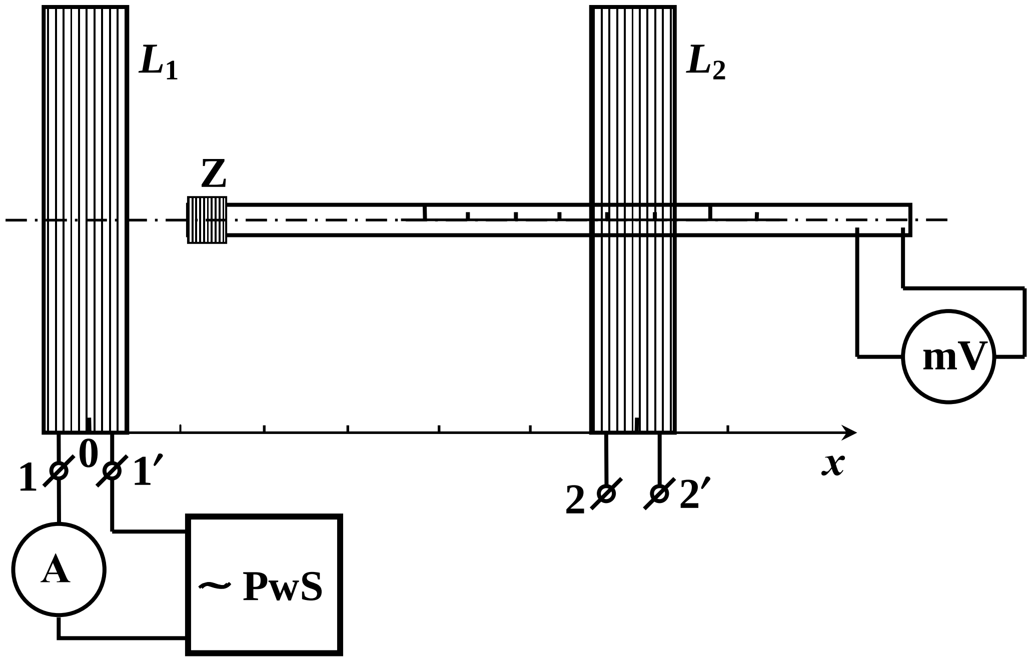


Figure 2.13.i

An AC current of $f = 50$ Hz frequency is used to produce an alternating magnetic filed by L_1 and L_2 coils. The magnitude of the field can be written as

$$B(t) = B_{\max}\cos(\omega t + \varphi_0),$$

where $B_{\max}$ is the peak value of magnetic induction, $\omega = 2\pi f$ is an angular frequency of AC current, and φ_0 is an initial phase shift. Thus,

$$\frac{dB}{dt} = -\omega B_{\max}\sin\left(\omega t + \varphi_0\right).$$

If the probe is located in the vicinity of the coil, the induced EMF is

$$\mathcal{E} = NA\omega B_{\max}\sin\left(\omega t + \varphi_0\right),$$

where $NA\omega B_{\max} = \mathcal{E}_0$ is the peak value of the induced electromotive force. The EMF of the probe is measured by an AC-millivoltmeter. It shows the effective value of the voltage:

$$U = \frac{\mathcal{E}_0}{\sqrt{2}} = \frac{NA\omega}{\sqrt{2}}B_{\max}.$$

Thus, by measuring the voltage U induced on the probe, one can find the peak value of the magnetic field at the position of the probe:

$$B_{\max} = \frac{\sqrt{2}}{NA\omega}U. \tag{2.13.1}$$

The expression of the magnetic field B on the axis of circular loop with current I can be obtained based on the Biot-Savart's law and the superposition principle:

$$B = \frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{3/2}}, \tag{2.13.2}$$

where R is the radius of the loop, x is the distance from the center of the loop to the observation point (Fig. 2.13.ii), $\mu_0 = 4\pi \cdot 10^{-7}$ H/m is the magnetic constant. Magnetic field at the center of the circular current loop (x = 0) is calculated as follows:

$$B_0 = \frac{\mu_0 I}{2R}. \tag{2.13.3}$$

Substituting B_0 from this equation into Eq. (2.13.2), the latter can be rewritten as

$$B = B_0 \frac{R^3}{\left(R^2 + x^2\right)^{3/2}} = \frac{B_0}{\left(1 + (x/R)^2\right)^{3/2}}. \tag{2.13.4}$$

If the coil length is much smaller than the diameter (short solenoid) and contains N_C loops that are closely spaced, then the magnetic field of such a coil is approximately equal to the magnetic field of a circular loop with the current N_C times higher than for one loop. Therefore at the axis of the coil, the magnetic field can be approximately determined by Eq. (2.13.4).

According to Eq. (2.13.1), the magnetic field $B(x)$ of the coil is proportional to the induced voltage $U(x)$ on the probe where x is

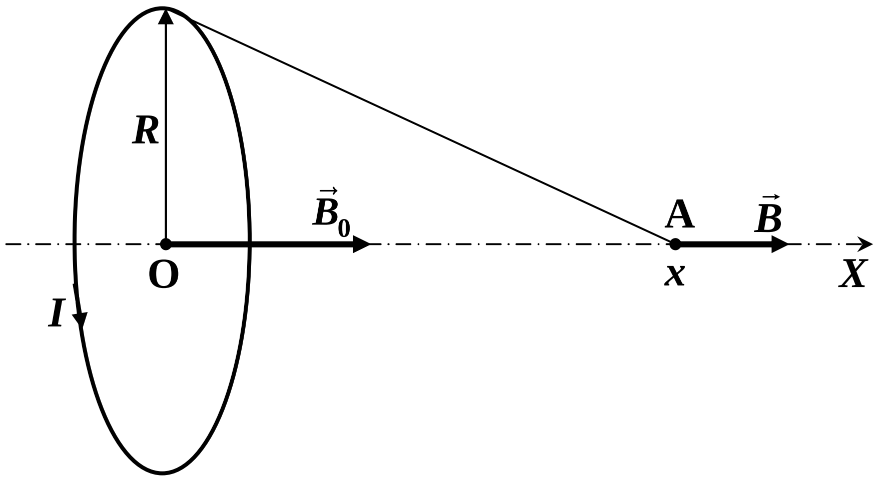


Figure 2.13.ii

the position of the probe. Thus, the relative value of the magnetic field $B(x)/B_0$ can be determined experimentally

$$\left(\frac{B(x)}{B_0}\right)_{exp} = \frac{U(x)}{U_0}. \qquad (2.13.5)$$

The theoretical expression for a relative magnetic field at the axis of the coil is represented by a function

$$f(x) = \left(\frac{B(x)}{B_0}\right)_{th} = \frac{1}{\left(1 + (x/R)^2\right)^{3/2}}. \qquad (2.13.6)$$

After comparing the results of the experimental (2.13.5) and theoretical (2.13.6) values of the relative magnetic field at the axis of the coil, one can conclude on the validity of assumptions.

We use two identical coils L_1 and L_2 to verify the superposition principle (see Fig. 2.13.i). The resulting magnetic field created by two coils with current is the following

$$\vec{\boldsymbol{B}} = \vec{\boldsymbol{B}}_1 + \vec{\boldsymbol{B}}_2, \qquad (2.13.7)$$

where $\vec{\boldsymbol{B}}_1$ and $\vec{\boldsymbol{B}}_2$ are fields of a coil L_1 and L_2 respectively.

If two coils are coaxial and the direction of the current is the same for both of them, then vectors $\vec{\boldsymbol{B}}_1$ and $\vec{\boldsymbol{B}}_2$ at the axis of

Table 2.13.i

x, cm	0	2	4	6	...	...	30
U_1, mV							
$\dfrac{B_1(x)}{B_0}$							
$f(x)$							

the system have the same direction. Thus, Eq. (2.13.7) can be rewritten for magnetic field magnitudes only

$$B(x) = B_1(x) + B_2(x)\,. \qquad (2.13.8)$$

This equation can be verified experimentally by measuring the magnetic fields of individual coils and the resulting field of two coils. The theoretical value calculated by Eq. (2.13.8) can be compared with the experimental one.

Step-by-step Procedure of the Experiment

Task 1. The magnetic field of a single coil.

1. Connect the power supply to the terminals " 1" and " 1′ " of the coil L_1 and establish an effective circuit current of $I = 1$ A.
2. Check the location of the probe (it should be located in the middle of the coil L_1 which corresponds to the coordinate $x = 0$ cm).
3. Measure probe voltage with a millivoltmeter $U_1(x)$.
4. Repeat measurements, shifting the probe along the axis of the coil with increments of 2 cm. Write the measured data into Table 2.13.i.
5. Calculate the corresponding relations $B_1(x)/B_0$ according to Eq. (2.13.5) where $B_1(0) = B_0$. Tabulate calculations at Table 2.13.i.

Table 2.13.ii

x, cm	0	2	4	6	...	...	30
U_2, mV							
$\dfrac{B_2(x)}{B_0}$							
U_{1+2}, mV							
$\dfrac{B_{1+2}(x)}{B_0}$							
$\dfrac{B_1(x)+B_2(x)}{B_0}$							

6. Calculate the corresponding values of the function $f(x)$ (Eq. 2.13.6) for all positions x from Table 2.13.i and put them onto the table.
7. Plot the dependences $B_1(x)/B_0$ and $f(x)$ vs. position x of the probe at the same graph. Compare the curves and make conclusions.

Task 2. The magnetic field of two coils. The superposition principle.

1. Place the coil L_2 at the distance specified by instructor from the first coil. Connect the power supply to the terminals " 2" and " $2'$ " of the coil L_2 (see Fig. 2.13.i). Coil L_1 should be disconnected.
2. Establish a circuit current of $I = 1$ A. Measure the voltage of the probe $U_2(x)$ and calculate the ratio $B_2(x)/B_0$ for coil L_2. Take into account that ratio is equal to 1 at the position of the coil L_2. Enter the data into Table 2.13.ii.
3. Disconnect the power supply.
4. Connect the coils L_1 and L_2 in serial. The direction of the current in the coils have to be the same. Connect the power

supply and establish the circuit current of 0.5 A.

5. Measure voltage $U_{1+2}(x)$ induced in a probe by alternating magnetic filed of both coils. To make results comparable to the data obtained in the first task, double the measured value of $U_{1+2}(x)$. Calculate ratio $\dfrac{B_{1+2}(x)}{B_0}$. Take into account that $B_0 = B(0)$. Write the data to Table 2.13.ii.
6. Calculate the sum of two fields of both coils $\dfrac{B_1(x) + B_2(x)}{B_0}$. Add the data to Table 2.13.ii.
7. Build the a plots of $\dfrac{B_1(x)}{B_0}$, $\dfrac{B_2(x)}{B_0}$, $\dfrac{B_{1+2}(x)}{B_0}$, and $\dfrac{B_1(x) + B_2(x)}{B_0}$ vs. position x of the probe at the same graph.
8. Compare theoretical and experimental curves of the resulting magnetic field. Make conclusions.

After-lab Questions

Case 1.

1. Write an expression for Biot-Savart law. Draw a picture to explain it.
2. Derive an expression for the magnetic field at the center of the current loop.
3. What phenomenon is used to measure the magnetic field in this experiment?
4. *Problem.* The long wire carrying a current of $I = 4$ A is bent at the angle $\phi = 120°$. Find the magnetic field at a distance $a = 5$ cm from the vertex of the angle on its bisector.
   Answer: $B = 2.4 \cdot 10^{-5}$ T.

Case 2.

1. State the superposition principle of fields.
2. Derive an expression for the magnetic field of an infinitely long solenoid.
3. How does the radius of the coil affect the magnetic field?

4. *Problem.* Current $I = 30$ A flows along a straight-line segment of wire with the length $l = 80$ cm. Find the magnetic field at a point equidistant from the ends of the segment and at a distance $r_0 = 30$ cm from the wire.
   Answer: $B = 1.6 \cdot 10^{-5}$ T.

Case 3.

1. Formulate Ampere's circulation law of the vector $\vec{\boldsymbol{B}}$. Draw a picture to explain it.
2. Derive an expression for calculating a magnetic field of a long straight current-carrying conductor.
3. Calculate the relative magnetic field at the axis of the system created by two coils if the separation distance between them is equal to their radius. Perform calculation at the position of the coils and in the middle between them.
4. *Problem.* A very long straight horizontal wire carries a current such that $5 \cdot 10^{20}$ electrons per second pass any given point going from west to east. What are the magnitude and direction of the magnetic field this wire produces at a point 40.0 cm directly above it?
   Answer: $B = 4 \cdot 10^{-5}$ T.

LABORATORY EXPERIMENT 2.15

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

Purpose of the Experiment: to investigate experimentally the phenomenon of electromagnetic induction and to verify Faraday's law.

Equipment and Accessories: solenoid with two coaxial windings, generator of sawtooth signal, double–channels oscilloscope.

Basic Methodology. The sawtooth voltage is applied to one of two coaxial coils. The electromotive force is induced in the sec-

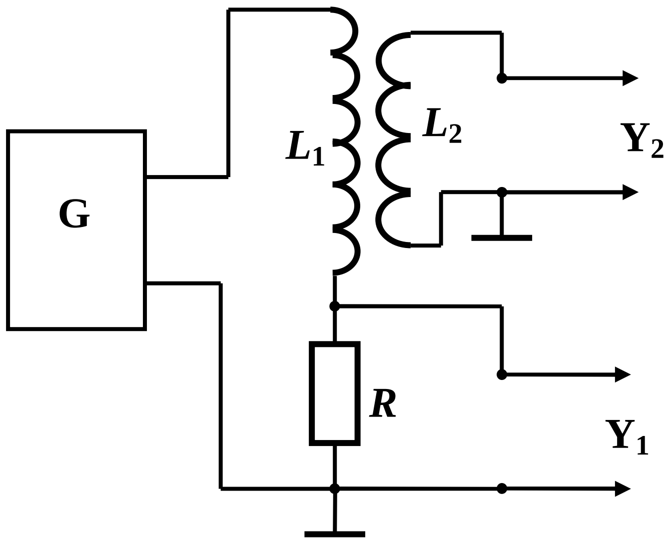


Figure 2.15.i

ond coil. Patten of this EMF is observed at the oscilloscope screen. Faraday's law is verified by comparing experimentally measured and theoretically calculated value of EMF induced in the second coil under different frequencies and amplitudes of the sawtooth signal.

Recommended Pre-lab Reading: [1] 29.2; [2] 30.1; [3] 31.1.

Pre-lab Questions

1. What is the SI unit of a magnetic flux?
2. Formulate Faraday's law of electromagnetic induction.
3. Formulate Lenz's law.

Theoretical Introduction

The diagram of installation used in this experiment is shown in Fig. 2.15.i. It consists of a solenoid with two coaxial coils L_1 and L_2. They are wound on the same hollow cylinder. Lengths l_1 and l_2 of the coils, number of turns N_1 and N_2, and diameter D of the cylinder are specified on the equipment.

Sawtooth voltage U_1 is applied to the coil L_1 from the generator **G**. The frequency of *teeth* is in a range $\nu = 500...1000$ Hz. Total resistance R of the circuit with the L_1 coil is 50 Ω. Magnetic field created by a current I_1 in the coil L_1 is:

$$B_1 = \mu_0 \frac{N_1 I_1}{l_1},$$

here $\mu_0 = 4\pi \cdot 10^{-7}$ H/m is a magnetic constant.

Mutual magnetic flux of the coil L_2 is created by the field B_1

$$\Psi_{m2} = N_2 B_1 A = \mu_0 \frac{N_1 N_2}{l_1} A I_1,$$

where A is the cross-sectional area of the solenoid.

According to Faraday's law, the EMF should be induced in the coil L_2 if the value of the mutual magnetic flux Ψ_{m2} varies in time:

$$\mathcal{E}_i = -\frac{d\Psi_{m2}}{dt} = -\mu_0 \frac{N_1 N_2 A}{l_1} \frac{dI_1}{dt}. \tag{2.15.1}$$

The current in the first coil is directly proportional to the voltage applied and reversely proportional to the resistance of the circuit:

$$I_1 = \frac{U_1}{R}.$$

Thus, Eq. (2.15.1) can be written as:

$$\mathcal{E}_i = -\mu_0 \frac{N_1 N_2 A}{l_1 R} \frac{dU_1}{dt}. \tag{2.15.2}$$

Faraday's law of electromagnetic induction can be verified by comparing the experimental value of the EMF induced in the coil L_2 to the one calculated by Eq. (2.15.2).

The graph of the voltage versus time is observed at the screen of the oscilloscope. The vertical input (Y_1) of the first channel of the oscilloscope is under the signal proportional to the voltage U_1 applied to the first coil. The EMF $\mathcal{E}_i$ induced on the coil L_2 is applied to the vertical input (Y_2) of the second channel. Since the

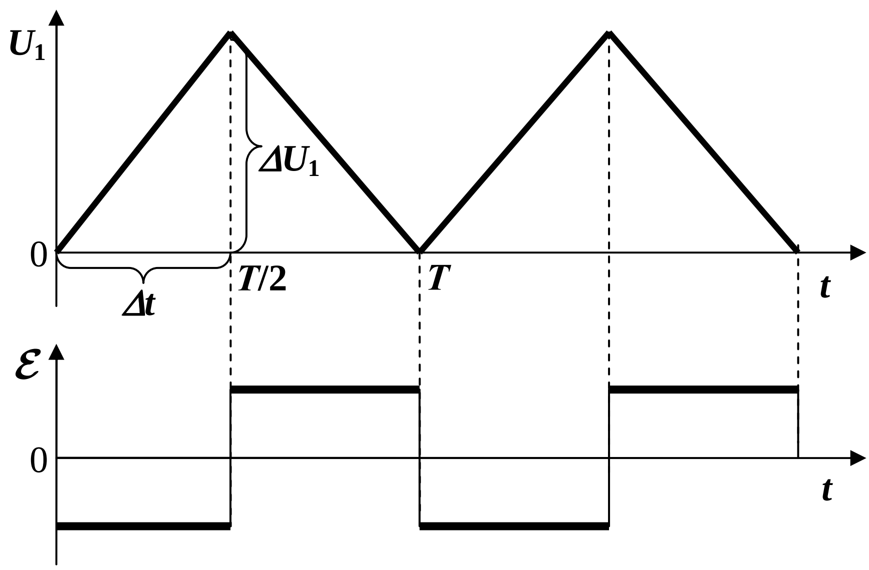


Figure 2.15.ii

time derivative of the sawtooth signal is a constant over the half of the period (Fig. 2.15.ii), Eq. (2.15.2) can be written as

$$\mathcal{E}_i = -\mu_0 \frac{N_1 N_2 A}{l_1 R} \frac{\Delta U_1}{\Delta t}. \tag{2.15.3}$$

Step-by-step Procedure of the Experiment

1. Verify connection of generator and oscilloscope to the coils (see Fig. 2.15.i). Turn on the power of all your laboratory equipment. Assign the frequency of the output voltage on generator $f_1 = 500$ Hz. Use oscilloscope to evaluate frequency.
2. Achieve a stable picture at the screen of oscilloscope, similar to Fig. 2.15.ii. Use the scale control handles over time axis t ("ms/g.p.") and voltage axis U ("V/g.p.") and handle "Stabilization".
3. Use the pattern at the oscilloscope screen to define values of ΔU_1, $\mathcal{E}_{exp1}$, and time interval Δt_1 (see Fig. 2.15.ii).
4. Substitute magnitudes ΔU_1 and Δt_1 to Eq. (2.15.3) and calculate the average theoretical value of $\langle \mathcal{E}_{th} \rangle_1$.
5. Compare values of $\mathcal{E}_{exp1}$ and $\langle \mathcal{E}_{th} \rangle_1$.

6. Repeat measurements with the oscillation frequency of the output voltage $f_2 = 1000$ Hz and the same amplitude of the voltage ΔU_1. Compare $\mathcal{E}_{exp2}$ and $\langle \mathcal{E}_{th} \rangle_2$.
7. Repeat measurements with the frequency $f_3 = f_1 = 500$ Hz of the sawtooth signal and double amplitude of the voltage ΔU_1. Compare the magnitudes of $\mathcal{E}_{exp3}$ and $\langle \mathcal{E}_{th} \rangle_3$.
8. Check the equality in the following ratios: $\dfrac{\mathcal{E}_{exp2}}{\mathcal{E}_{exp1}}$ and $\dfrac{\Delta t_1}{\Delta t_2}$; $\dfrac{\mathcal{E}_{exp1}}{\mathcal{E}_{exp3}}$ and $\dfrac{\Delta U_1}{\Delta U_3}$. Make conclusions.

After-lab Questions

Case 1.

1. Provide the definition of the magnetic flux. Write the expression of the magnetic flux through an arbitrary surface.
2. Formulate the Lenz's law.
3. Why is the sawtooth signal applied to a first coil?
4. *Problem.* The EMF πt (in SI units) is induced in a circular loop placed in a uniform magnetic field that varies with time. The radius of the loop is 1 m. The magnetic field lines make an angle of 60° with the normal to the plane of the loop. Find magnetic field $B(t)$ as a function of time, given that it started from zero ($B(0) = 0$).
   Answer: $B = t^2$.

Case 2.

1. A loop of wire is placed in a uniform magnetic field. For what orientation of the loop is the magnetic flux a maximum? For what orientation is the flux zero?
2. Formulate and write Gauss's law for magnetic field. What information is unhidden by this law about magnetic field?
3. Why the induced EMF is negative when the voltage in the first coil is increasing?

4. *Problem.* The coil of diameter $D = 8$ cm and $N = 80$ turns is placed in a uniform magnetic field $B = 0.06$ T. The coil is turned to an angle of 180° in $\Delta t = 0.2$ s, and its axis remains parallel to the magnetic field lines. Find an average EMF induced in the coil.
   Answer: $\mathcal{E}_i = 0.24$ V.

Case 3.

1. Formulate the Lens's law.
2. Why is the magnetic flux through an arbitrary closed surface equal to zero?
3. Why do the square-wave pulses arise in the second coil if the sawtooth signal is applied to a fist coil?
4. *Problem.* The circular loop of radius $r = 4$ cm and resistance $R = 3.14\ \Omega$ is placed in a uniform magnetic field $B = 0.125$ T. The angle between the plane of the loop and field lines is 30°. What charge will be induced in the loop if magnetic field disappears?
   Answer: $q = 10^{-4}$ C.

LABORATORY EXPERIMENT 2.16

DETERMINATION OF THE ANGLE OF MAGNETIC INCLINATION

Purpose of the Experiment: to determine the angle of magnetic inclination and the value of the magnetic field of the Earth by using ballistic method.

Equipment and Accessories: inductor **I**; ballistic galvanometer **G**.

Basic Methodology. An inductor coil can be rotated about horizontal and vertical axes. A change of the Earth magnetic field flux through the coil causes an induced current in it. Current magnitude and, therefore, amount of charge passed through the circuit, are proportional to the corresponding component of the Earth magnetic field. By measuring the passed charge for two ways of rotation, horizontal and vertical components of the Earth magnetic field and the angle of magnetic inclination can be calculated.

Recommended Pre-lab Reading: [1] 27.1, 29.1, 29.2, 29,3; [2] 28.3, 30.1; [3] 31.1, 31.3.

Pre-lab Questions

1. Define the magnetic flux. What is the expression for magnetic flux through a plane surface in a uniform magnetic field?
2. What is a magnetic field line? What is the direction of these lines?
3. Formulate Faraday's Law.

Theoretical Introduction

Earth has a magnetic field like that of a huge bar magnet. Earth's north geographic pole is close to a magnetic south pole that is why the north pole of a compass needle points north and magnetic field lines are directed from south to north (Fig. 2.16.i). The Earth magnetic axis is not quite parallel to its geographic axis (the axis of rotation), so a compass reading deviates somewhat from the geographic north. This deviation which varies with location is called *magnetic declination*, or *magnetic variation.* Also, the magnetic field is not horizontal at most points on the Earth surface; its angle (up or down) is called *magnetic inclination.* At the magnetic poles, the magnetic field is vertical and the angle of magnetic inclination is 90°.

At each point, magnetic field of the Earth $\vec{\boldsymbol{B}}_E$ can be decomposed into vertical $\vec{\boldsymbol{B}}_v$ and horizontal $\vec{\boldsymbol{B}}_h$ components: $\vec{\boldsymbol{B}}_E = \vec{\boldsymbol{B}}_v + \vec{\boldsymbol{B}}_h$. In Fig. 2.16.ii, the angles of inclination, φ, and

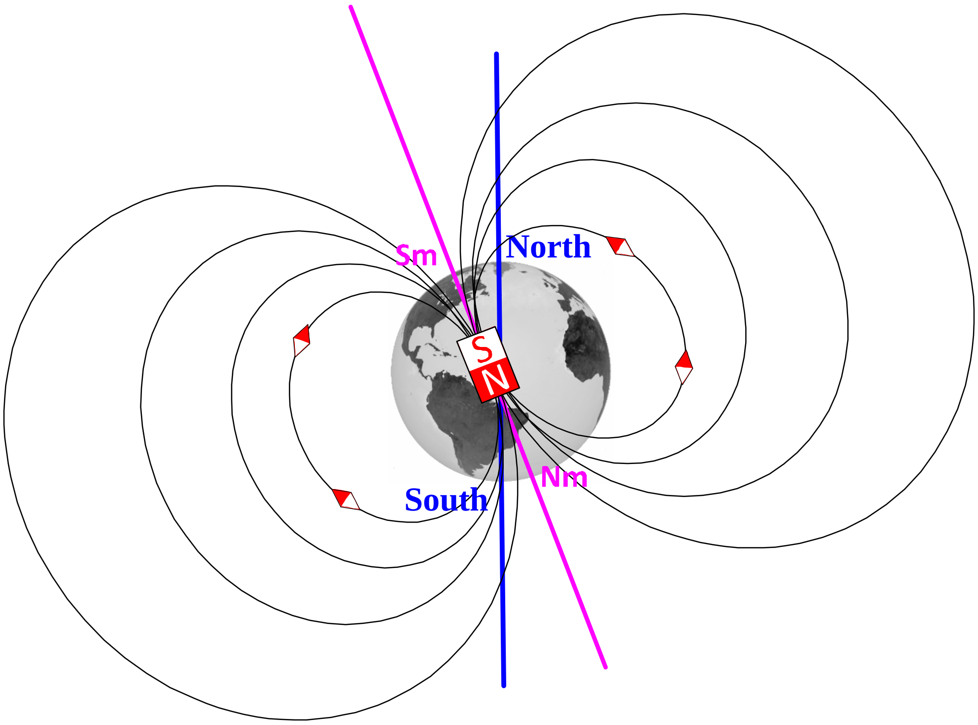


Figure 2.16.i

declination, θ, are shown. If vertical and horizontal components are known, the angle of inclination φ can be computed as

$$\varphi = \arctan \frac{B_v}{B_h}. \tag{2.16.1}$$

In this experiment, a modification of the ballistic method, based on electromagnetic induction phenomenon, is used to measure magnitudes of the vertical $\vec{\boldsymbol{B}}_v$ and horizontal $\vec{\boldsymbol{B}}_h$ magnetic field components. The setup (Fig. 2.16.iii) contains an inductor **I** and a ballistic galvanometer **G**. The inductor consists of a coil (1), current-collecting bushings (2), stanchions (3), and a platform (4).

By using bushings and stanchions, the coil is mounted on the platform and can be rotated freely. To make measurements easier, the rotation axis passes through the center of mass of the coil along its diameter. The coil has N turns of wire, the ends of the wire connected to the terminals (5) which are in their turn connected to the galvanometer **G**. Under rotation about the axis during the time interval Δt, there is a change in Earth magnetic field flux Φ_m

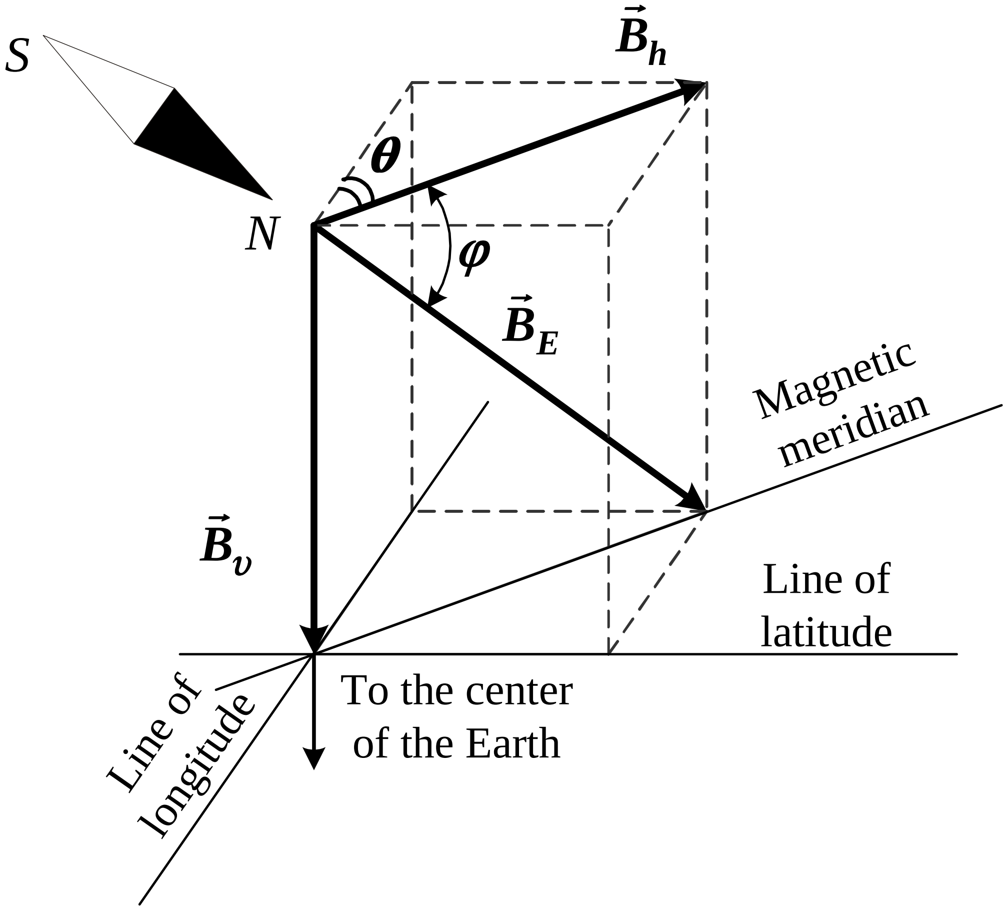


Figure 2.16.ii

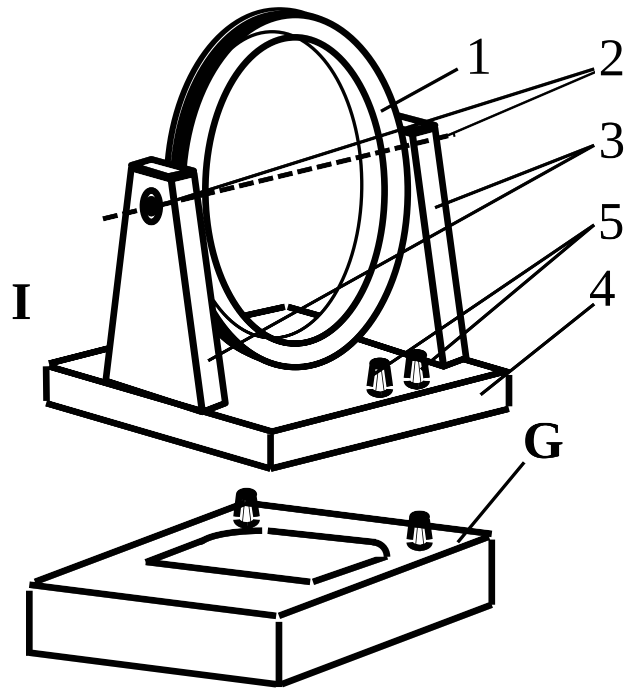


Figure 2.16.iii

through each turn of the coil. Thus, there is a change in the net flux through the coil $\Psi_m = N\Phi_m$ by the amount $\Delta\Psi_m = \Psi_f - \Psi_i$, where Ψ_i is the initial value of net flux, and Ψ_f is the final one. According to Faraday's law, the change of magnetic flux through a closed conductive circuit induces an electromotive force (EMF) $\mathcal{E}_i$ in the circuit and the EMF is proportional to the rate of change of the net flux through the coil:

$$\mathcal{E}_i = -\frac{d\Psi_m}{dt}.$$

EMF $\mathcal{E}_i$ induces the current in the coil:

$$I_i = \frac{\mathcal{E}_i}{R} = -\frac{d\Psi_m}{Rdt},$$

where R is the circuit resistance. Since,

$$I = \frac{dq}{dt},$$

then:

$$\frac{dq}{dt} = -\frac{d\Psi_m}{Rdt}.$$

The amount of charge dq flowing through the circuit per infinitesimal time interval dt is

$$dq = Idt = -\frac{d\Psi_m}{R}.$$

Integrating the above expression for dq over the time interval during which the magnetic flux changes from Ψ_i to Ψ_f, we have a total amount of charge passed through the circuit:

$$q = -\int\limits_{\Psi_{mi}}^{\Psi_{mf}} \frac{d\Psi_m}{R} = \frac{\Psi_{mi} - \Psi_{mf}}{R} = \frac{\Delta\Psi_m}{R}. \tag{2.16.2}$$

In this experiment, a net flux change through the inductor $\Delta\Psi_m$ is caused by the change in the coil orientation relative to the magnetic field lines, i.e. a change of the angle between the normal to

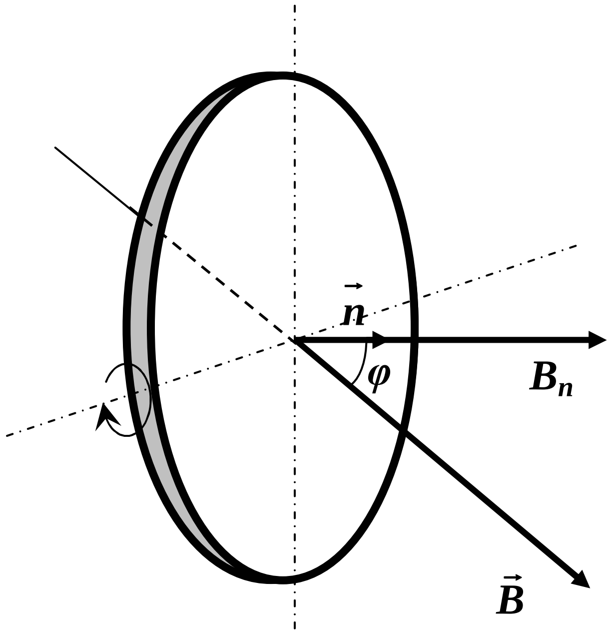


Figure 2.16.iv

the coil plane and the magnetic field. Therefore, if the inductor coil is flipped over (made a 180° turn), a new value of the net flux will be opposite in sign, but its magnitude will not change. Then,

$$\Delta\Psi_m = 2\Psi = 2B_n AN, \tag{2.16.3}$$

where B_n is a projection of the $\vec{\boldsymbol{B}}_E$ onto the direction of the normal to the coil plane in initial state; A is an area of the coil; N is the number of turns in the inductor coil (Fig. 2.16.iv).

If the initial position of the coil is such that its normal $\vec{\boldsymbol{n}}$ is vertical, B_n is the vertical projection of the Earth magnetic field B_v. If $\vec{\boldsymbol{n}}$ is horizontal, B_n is the horizontal projection of the Earth magnetic field B_h.

After substituting (2.16.3) in (2.16.2), we obtain that the amount of charge flowing through the coil at its rotation through 180° depends on the magnetic field component which is perpendicular to the plane of the coil initial position. Therefore, by measuring the charge, the magnitude of this component can be determined:

$$B_{h,v} = \frac{\Delta\Psi_m}{2AN} = \frac{qR}{2AN}. \tag{2.16.4}$$

When the rotation axis is vertical (the plane of the coil initial position is vertical too), component B_h is calculated. And when the rotation axis is horizontal (the plane of the coil initial position is horizontal too), component B_v is calculated.

The charge q, flowing through the coil due to electromagnetic induction phenomenon, is determined by using a ballistic galvanometer. Ballistic galvanometer differs from other moving-coil galvanometers in the way that the moment of inertia of its moving coil is increased, therefore, it has a larger period of oscillation. If the time of charge flowing through the galvanometer coil is much less than the period of the coil oscillation, then the magnitude of the galvanometer spot light first throw n is proportional to the charge that passed through the galvanometer:

$$q = Cn, \tag{2.16.5}$$

where C is galvanometer constant. Substituting Eq. (2.16.5) into (2.16.4), we obtain

$$B_h = \frac{CR}{2NA} n_h = \beta n_h, \tag{2.16.6}$$

where β is a constant of the given setup (specified at the equipment). It is clear that a similar relationship is valid for B_v:

$$B_v = \beta n_v. \tag{2.16.7}$$

Step-by-step Procedure of the Experiment

1. Connect the ballistic galvanometer to the terminals of the inductor and plug in the galvanometer.
2. Measure the magnitude of the Earth magnetic field vertical component B_v. For this:
   (a) place the inductor so that the coil rotation axis has horizontal direction along the geographic meridian (the meridian direction is indicated at the workplace), and the coil plane is horizontal;

Table 2.16.i

#	Measurement of B_v			Measurement of B_h			φ, deg.	B_E, μT
	n_{vert}, g.p.	$\langle n_{vert} \rangle$, g.p.	B_v, μT	n_{hor}, g.p.	$\langle n_{hor} \rangle$, g.p.	B_h, μT		
1								
2								
...								
10								

(b) make a quick 180° turn of the coil and read the absolute value of the galvanometer spot light first throw n_1;

(c) quickly return the coil to its initial position and read the absolute value of the galvanometer spot light throw n_2;

(d) repeat the measurements (b), (c) five times each and calculate the average $\langle n_{vert} \rangle$ (over 10 measurements n_i). Write all the data into Table 2.16.i.

3. Measure the magnitude of the Earth magnetic field horizontal component B_h. For this, place the coil so that its rotation axis is vertical and the coil plane is perpendicular to the geographic meridian plane. Make the same measurements as in Steps 2 (b, c, d), and calculate the average $\langle n_{hor} \rangle$.
4. Calculate the values B_v and B_h using the Eqs. (2.16.6) and (2.16.7).
5. Calculate the angle of magnetic inclination φ by using (2.16.1).
6. Calculate the magnitude of the Earth magnetic field B_E using its known components B_v and B_h.

After-lab Questions

Case 1.

1. Formulate Faraday's law of electromagnetic induction. Formulate Lenz's law.

2. Explain of the method of magnetic inclination angle determination by using inductor.
3. What physical quantity can be measured by using a galvanometer? Why is it a ballistic galvanometer that is used in this experiment?
4. *Problem.* Circular loop of radius $r = 2$ cm and resistance of $R = 3.14\ \Omega$ is located in a uniform magnetic field of $B = 0.25$ T. Loop plane is perpendicular to the magnetic field. What charge will flow through the loop if one makes it a 90° turn?
   Answer: $q = 100\ \mu$C.

Case 2.

1. Draw the Earth magnetic field. In your drawing indicate the location of the Earth geographical and magnetic poles, and the angle of magnetic inclination.
2. Explain the nature of an induced EMF when there is a changing magnetic flux through a stationary conductor.
3. Describe the self-induction phenomenon. Write the formula for self-induced EMF.
4. *Problem.* A coil has the inductance $L = 0.02$ H and carries a time-varying current: $I(t) = I_0 \sin \omega t$ where $I_0 = 5$ A, $\omega = 300$ rad/s. Find the time dependence of the self-induced EMF.
   Answer: $\mathcal{E} = -30 \cos 300t$.

Case 3.

1. Describe the phenomenon of electromagnetic induction. How is it used in this experiment?
2. Derive an expression for calculating the charge that passed through the conducting loop under its rotation in magnetic field.
3. What peculiarities do magnetic field lines have? Draw a field produced by a circular loop, a long solenoid; show their direction. In what case is there a uniform magnetic field?

4. *Problem.* Copper hoop of mass $m = 5$ kg is located in the plane of magnetic meridian. What charge will flow through the hoop when it is rotated about its vertical axis by the angle of 90°? The value of the Earth magnetic field horizontal component is $B_h = 20\ \mu$T. The mass density of copper is $\rho = 8900$ kg/m^3, and its the electrical conductivity $\sigma = 5.8 \cdot 10^7\ \Omega^{-1} \cdot$ m^{-1}.
   Answer: $q = 52$ mC.

LABORATORY EXPERIMENT 2.18

ANALYSIS OF SELF-INDUCTION PHENOMENON

Purpose of the Experiment: to investigate processes of the current growth and decay in a circuit containing inductor and to determine the time constant.

Equipment and Accessories: oscilloscope with a rectangular pulse generator; RL–circuit.

Basic Methodology. RL–circuit is powered by a square-wave generator. Time constant is determined based on the pattern of current growth and decay observed at the oscilloscope screen.

Recommended Pre-lab Reading: [1] 30.4; [2] 32.4; [3] 32.2.

Pre-lab Questions

1. What is the SI unit for inductance?
2. Make a definition of the circuit inductance.
3. What is the time constant of RL–circuit.

Theoretical Introduction

If a circuit contains an inductor such as coil, the self-inductance of the coil prevents the current in the circuit from increasing or decreasing instantaneously.

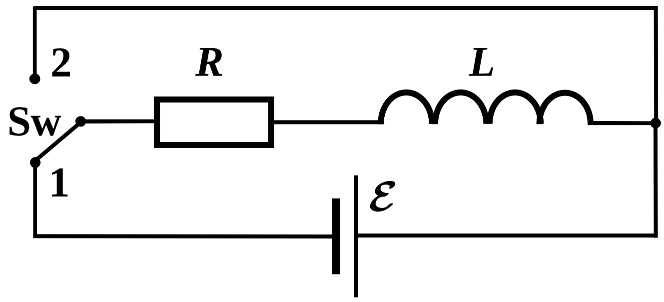


Figure 2.18.i

The circuit diagram for examining the current growth and decay in RL–circuits is shown in Fig. 2.18.i. It consists of a battery of constant electro-motive force $\mathcal{E}$, an ideal coil of inductance L, and a resistance R in series. The internal resistance of the battery and all connecting wires is neglected. By using a switch **Sw**, the coil can be connected to the battery (position 1, the current growth) or disconnected from it (position 2, the current decay).

The expressions are derived below for time dependence of a current in a RL–circuit for current growth and decay. Let's consider current growth through the inductor. Suppose that the battery has been initially disconnected from the circuit (switch **Sw** in a position 2). The circuit has no current in it. When the circuit with the battery and the coil is closed (**Sw** is in position 1), there is a time-varying current in the circuit. According to Faraday's law, the EMF of self-induction is induced in the inductor:

$$\mathcal{E}_{si} = -L\frac{dI}{dt}. \tag{2.18.1}$$

According to Ohm's law for a closed loop, the potential difference across the resistor R is equal to the total EMF on the circuit:

$$IR = \mathcal{E} + \mathcal{E}_{si}, \tag{2.18.2}$$

Taking into account Eq. (2.18.1), Eq. (2.18.2) can be rearranged as follows:

$$\frac{dI}{dt} = -\frac{R}{L}\left(I - \frac{\mathcal{E}}{R}\right). \tag{2.18.3}$$

This differential equation can be solved by the variable separation method. Taking into account that the initial current is zero, one can easily derive the behavior of the current as a function of time:

$$I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{R}{L}t}\right), \qquad (2.18.4)$$

here $e \approx 2.71828$ is a basis of natural logarithm.

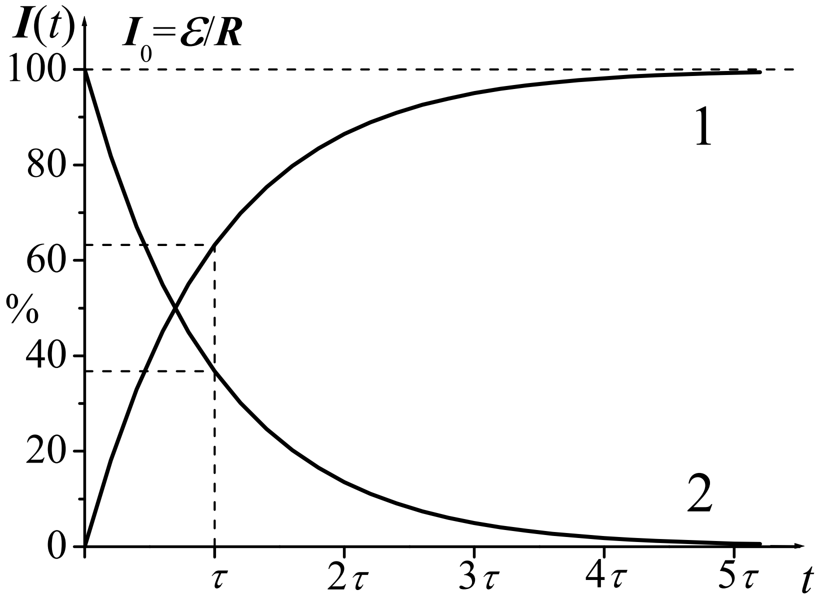


Figure 2.18.ii

Graph of I versus t for the current growth in the RL–circuit with EMF in series is shown in Fig. 2.18.ii (curve 1). The value of the instantaneous current I rises rapidly at beginning. Then it increases slower and approaches the final value $I_0 = \mathcal{E}/R$ asymptotically. At a time equal to L/R, the current has risen to $(1-1/e)$, or about 63%, of its final value. The quantity L/R is therefore a measure of how quickly the current builds toward its final value. This quantity is called the time constant for the circuit, or relaxation time, denoted by τ:

$$\tau = \frac{L}{R}. \qquad (2.18.5)$$

In a time equal to 2τ, the current reaches 86 % of its final value; in 3τ, 95 %; and in 5τ, 99.3 %. Thus, the current in an inductor never changes instantaneously, but after the current settles down to a constant value, the inductor plays no role in the circuit.

Now suppose that the switch **Sw** of the circuit in Fig. 2.18.i has been in a position 1 for a while, and the current has reached the value I_0. Resetting stopwatch to redefine the initial time, the switch **Sw** is thrown to the position 2 at time $t = 0$, bypassing the battery. The current through R and L does not instantaneously go to zero but decays smoothly. Again, the EMF of self-induction $\mathcal{E}_{si}$ (eq. (2.18.1)) is induced in the inductor. At the moment when the switch is repositioned, the battery with permanent EMF $\mathcal{E}$ is disconnected from RL–circuit. Hence, in this situation, Eq. (2.18.2) is equivalent to

$$IR = \mathcal{E}_{si} = -L\frac{dI}{dt}. \tag{2.18.6}$$

The solution of this equation can be found by the same variable separation method. Taking into account the initial condition $I(t)\Big|_{t=0} = I_0$, one can easily show that the current passing through the inductor varies with time according to

$$I(t) = I_0 e^{-\frac{R}{L}t}. \tag{2.18.7}$$

Graph of I versus t for the current decay in RL–circuit is shown in Fig. 2.18.ii (curve 2). The time constant, $\tau = L/R$, is the time for current to decrease to $1/e$, or about 37 %, of its original value. In time 2τ, it has dropped to 13.5 %, in time 3τ to 5 %, and in 5τ to 0.67 %. The time constant is a useful parameter for comparing the time responses of various circuits.

The scheme of the laboratory facility used in this experiment is shown in Fig. 2.18.iii. Connection to and disconnection from the battery is imitated by rectangular pulsing from the generator to

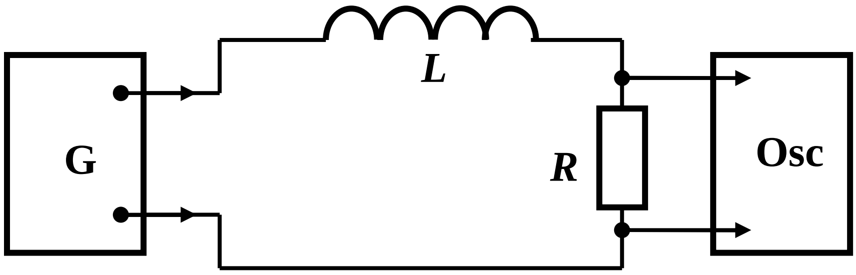


Figure 2.18.iii

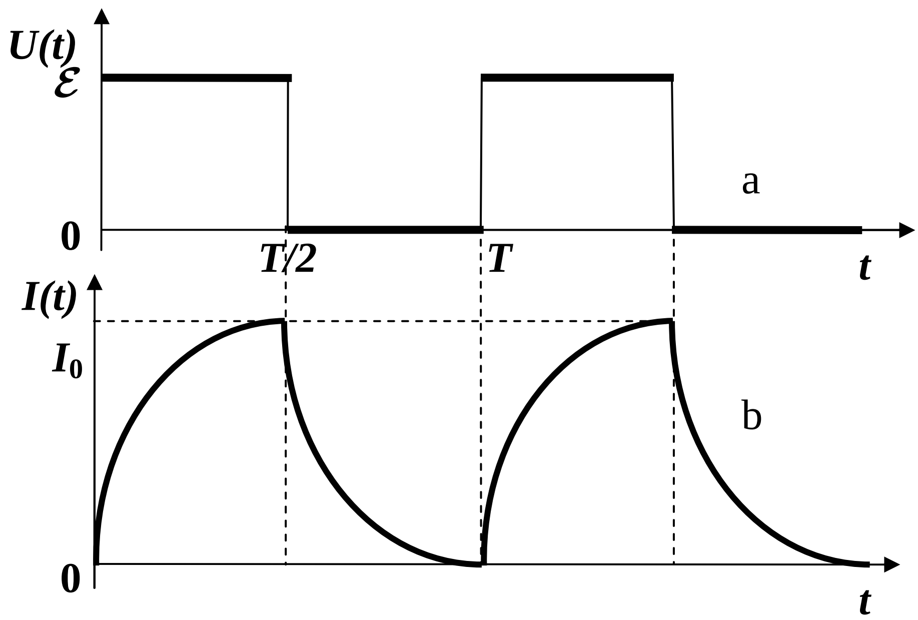


Figure 2.18.iv

the RL–circuit (Fig. 2.18.iv,a). Square-wave generator of rectangular pulses is built into the oscilloscope. There is a current going through the inductor L and the resistor R. The potential difference across the resistor V_R is applied to the oscilloscope input. According to Ohm's law ($V_R = I(t)R$), this voltage is proportional to current (Fig. 2.18.iv,b). The time constant of RL–circuit can be evaluated based at the time dependence of $I(t)$ observed on the oscilloscope screen.

For quantitative evaluation of time dependence of a current we can use the oscilloscope mode of operation when a continuous curve is shown as a dashed line (the mode "Marks", "Метки" in Russian). This technique with applying marks was described be-

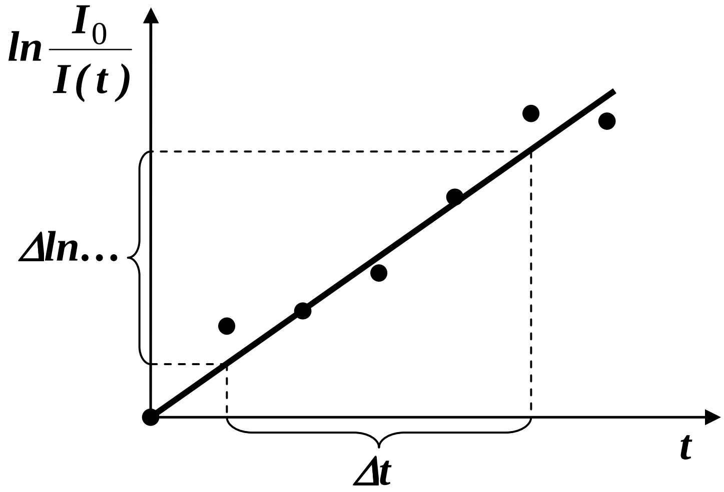


Figure 2.18.v

fore (Laboratory experiment 2.4). The time constant of RL-circuit is extracted from time dependence of a current by *the logarithmic method*. To apply this method, one has to plot graphs of the dependence of $\ln \dfrac{I_0}{I_0 - I(t)}$ (for current growth) and $\ln \dfrac{I_0}{I(t)}$ (for current decay) on time t (Fig. 2.18.v). The value of τ is determined by using an appropriate graph.

From Eqs. (2.18.4), (2.18.5), and (2.18.7), one can derive the following expressions for experimental evaluating the time constant:

$$\tau = \frac{\Delta t}{\Delta \ln \dfrac{I_0}{I_0 - I(t)}} \quad \text{(for current growth);} \tag{2.18.8}$$

$$\tau = \frac{\Delta t}{\Delta \ln \dfrac{I_0}{I(t)}} \quad \text{(for current decay).} \tag{2.18.9}$$

Δt is an arbitrarily chosen time interval, and the change of functions ($\Delta\left(\ln \dfrac{I_0}{I_0 - I(t)}\right)$ – for current growth or $\Delta\left(\ln \dfrac{I_0}{I(t)}\right)$ – for current decay) takes place within this time interval.

In the case, the dependences $\ln\frac{I_0}{I_0-I(t)}$ for current growth and $\ln\frac{I_0}{I(t)}$ for current decay are not linear; they have to be extrapolated by linear dependence (see Fig. 2.18.v). The experimental values of time constant τ_{exp} is defined by Eqs. (2.18.8) and (2.18.9) for current growth and decay respectively.

Step-by-step Procedure of the Experiment

Task 1. Current growth in the RL–circuit.

1. Check the connection of generator and oscilloscope to the RL–circuit and switch them on.
2. Switch on the oscilloscope (it should warm up for 2 – 3 minutes). Get a stable picture at the oscilloscope screen like in Fig. 2.18.iv,b. Set the time interval Δt_m between the adjacent marks according to the instructor's recommendation.
3. Determine time coordinates of the mark on the curve of the current growth by the formula $t_k = k\Delta t_m$. Determine the values of the ordinates I_k of the marks beginnings on the growing curve $I(t)$ and the maximum value of current I_0 (see Fig. 2.18.iv,b). Write the data into Table 2.18.i.

Table 2.18.i

Label $\#, k$	$t = k\Delta t_m, \mu$s	I_k, g.p.	$\dfrac{I_0}{I_0 - I_k}$	$\ln\dfrac{I_0}{I_0 - I_k}$	τ_{exp}, μs
0	0	0	1	0	
1					
2					
3					
4					
5					

4. Calculate the ratio $\frac{I_0}{I_0-I_k}$ and its logarithm for all values of the found current.

5. Construct the dependence graph of $\ln\frac{I_0}{I_0 - I(t)}$. Using it and Eq. (2.18.8), evaluate the time constant of this RL–circuit for current growth.

Task 2. Current decay in the RL–circuit.

1. Determine time coordinates of the marks on the curve of the current decay by the formula $t_k = k\Delta t_m$.
2. Determine the values of the ordinates I_k of the mark beginnings on the decaying curve $I(t)$. Write the data into Table 2.18.ii.
3. Calculate the ratio $\frac{I_0}{I_k}$ and its logarithm for all values of the found current.
4. Construct the dependence graph $\ln\frac{I_0}{I(t)}$. Using it and Eq. (2.18.9), evaluate the time constant of this RL–circuit for current decay.

Table 2.18.ii

Label #, k	$t = k\Delta t_m$, μs	I_k, g.p.	$\frac{I_0}{I_k}$	$\ln\frac{I_0}{I_k}$	τ_{exp}, μs
0	0		1	0	
1					
2					
3					
4					
5					

After-lab Questions

Case 1.

1. Write Ohm's law for close circuit that contains an inductor and a resistor connected in serial to DC battery. Explain it.

2. Derive the inductance of a toroid with cross-sectional area A, mean radius r and N closely wound turns of wire.
3. Does the time constant τ of the RL–circuit change due to the introduction of the ferromagnetic core inside the inductor? If it does, try to explain why this happens and how (becomes longer or shorter).
4. *Problem.* There is a current $I = I_0 \sin \omega t$ going through the coil of inductance $L = 0.021$ H. Here, $I_0 = 5$ A, $f = 50$ Hz. Find time dependence of the EMF induced in the circuit.
   Answer: $\mathcal{E} = 33 \sin (100\pi t - \pi/2)$ V.

Case 2.
1. Formulate Faraday's law. Provide examples.
2. Derive the time dependence of a current in the RL–circuit under current growth conditions.
3. Prove that the time constant τ of RL–circuit has the dimension of time.
4. *Problem.* A coil of inductance $L = 3$ mH and resistance $R = 150\ \Omega$ is connected to the DC power source. How much time does it take the current in coil to achieve half of maximum value?
   Answer: $\Delta t = 13.86\ \mu$s.

Case 3.
1. Write an expression for a magnetic field energy and magnetic field energy density.
2. Derive the time dependence of a current in the RL–circuit under current decay conditions.
3. What parameters of the circuit influence the rate of the current growth/decay?
4. *Problem.* All of the coil geometry were halved. The numbers of coil wraps and current remain the same. How does inductance change? How does magnetic field energy change?
   Answer: Both halved.

Chapter 3

WAVE AND QUANTUM OPTICS

LABORATORY EXPERIMENT 3.1

DETERMINATION OF THE REFRACTIVE INDEX OF GLASS USING THE INTERFERENCE METHOD

Purpose of the Experiment: to determine the refractive index of glass using the phenomenon of light interference in a glass plane-parallel plate.

Equipment and Accessories: optics bench, ruby laser LG72 with power supply IP-13, opaque screen with a short-focus lens, glass plane-parallel plate.

Basic Methodology. Light from a ruby laser source is reflected by two opposite surfaces of the glass plane-parallel plate and rings as interference fringes are observed on the screen. The diameters of two not adjacent rings are measured.

Recommended Pre-lab Reading: [1] 35.1, 35.4; [2] 35.1, 35.3, 37.3; [3] 35.4, 35.5, 37.4, 37.5.

Pre-lab Questions

1. What is the meaning of the index of refraction?
2. What happens when two waves combine, or interfere, in space?
3. What conditions must be met for constructive and destructive interference to occur?

Theoretical Introduction

When a parallel monochromatic beam of light 1 (Fig. 3.1.i) of the wavelength λ falls on the plane-parallel plate of thickness h, the plate reflects two parallel beams of light. The beam 2 is reflected by the upper surface of the plate, and the beam $2'$ is reflected by the lower one. Beams 2 and $2'$ are coherent as they come from the same source of light and they have an optical path difference. To calculate it let's drop a perpendicular from the point C onto the beam 2. The optical path difference between the beams 2 and $2'$ is

$$\Delta = (AB + BC)n - \left(AD - \frac{\lambda}{2}\right),$$

where n is the refractive index of the plate (for air $n = 1$). Here, an extra phase difference π occurs for the ray reflected from the upper surface of the plate because in this reflection, the incident beam goes from the rarer medium to the denser one. Therefore, the optical path of the beam 2 has decreased by $\lambda/2$. From Fig. 3.1.i, we can see that $AB = BC = h/\cos\beta$. According to the law of the light refraction

$$\sin\alpha = n\sin\beta \tag{3.1.1}$$

we have

$$AD = 2h\tan\beta\sin\alpha = 2hn\frac{\sin^2\beta}{\cos\beta}.$$

Then,

$$\Delta = \frac{2hn}{\cos\beta} - \frac{2hn\sin^2\beta}{\cos\beta} + \frac{\lambda}{2} = \frac{2hn}{\cos\beta}(1-\sin^2\beta) + \frac{\lambda}{2} = 2hn\cos\beta + \frac{\lambda}{2}. \tag{3.1.2}$$

Using (3.1.1) we can rewrite Eq. (3.1.2) as a dependence on the angle of incidence α:

$$\Delta = 2h\sqrt{n^2 - \sin^2\alpha} + \frac{\lambda}{2}. \tag{3.1.3}$$

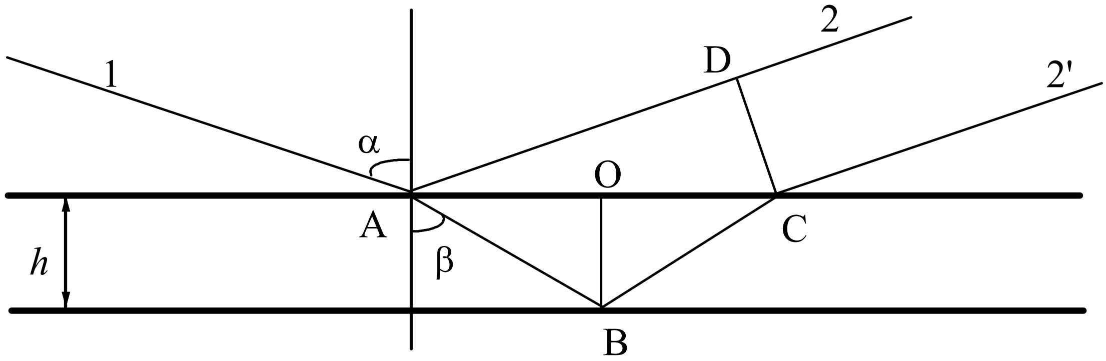


Figure 3.1.i

When Δ is an integer number of wavelengths, we expect to see constructive interference and a bright area and, when it is half-integer, destructive interference and a dark area. Therefore, the expression for maxima of the light reflected from the flat parallel plate is

$$2h\sqrt{n^2 - \sin^2\alpha} + \frac{\lambda}{2} = 2m\frac{\lambda}{2},$$

or

$$2h\sqrt{n^2 - \sin^2\alpha} = (2m-1)\frac{\lambda}{2},$$

or, in terms of the angle of refraction β,

$$2hn\cos\beta = (2m-1)\frac{\lambda}{2}, \tag{3.1.4}$$

where m is an integer ($m = \pm 1, \pm 2, \ldots$).

And the expression for the minima of the reflecting light is

$$2h\sqrt{n^2 - \sin^2\alpha} + \frac{\lambda}{2} = (2m+1)\frac{\lambda}{2},$$

or

$$2h\sqrt{n^2 - \sin^2\alpha} = 2m\frac{\lambda}{2},$$

or

$$2hn\cos\beta = 2m\frac{\lambda}{2}, \tag{3.1.5}$$

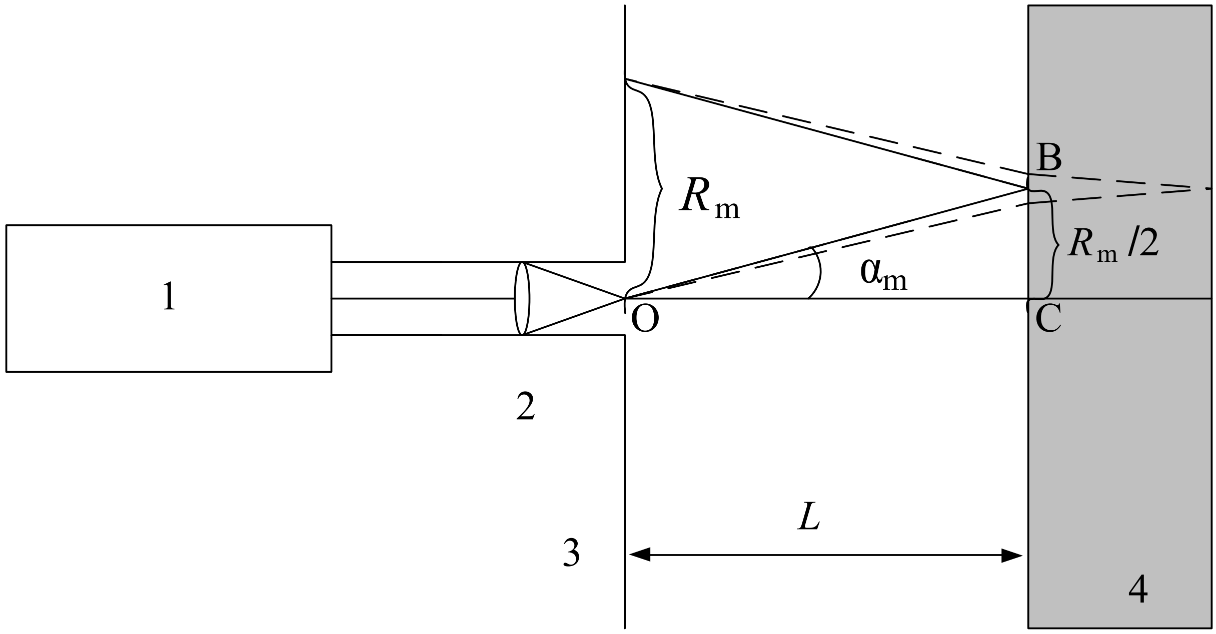


Figure 3.1.ii

where m is an integer ($m = \pm 1, \pm 2, \ldots$).

If the surfaces of the plate are perfectly parallel, the plate is bright or dark depending on the angle of incidence of the monochromatic light.

If a divergent beam (spherical wave) falls on the plate, its various rays have different angles of incidence. A pair of two rays reflected from the upper and lower surfaces corresponds to every incident ray. Optical path difference for every pair is expressed by Eq. (3.1.3), and it isn't the same for all points of the surface in respect to different angles of incidence.

Description of the Equipment

The scheme of laboratory facility is represented in Fig. 3.1.ii, where 1 is a laser; 2 is a short-focus lens; 3 is a screen; 4 is a plane-parallel plate.

To observe the interference in reflecting light the converging lens 2 (see Fig. 3.1.ii) and the screen 3, located in the lens focal plane, are used. After lightening the flat parallel plate by monochromatic light, the interference of reflecting light (reinforcement or cancellation) at different points on the screen depends only on the angle of light incidence on the plate. Thus, the interference pattern on the screen is a succession of bright and dark bands, or interference fringes. Every fringe corresponds to the definite angle of incidence. Therefore, such a pattern is called fringes of equal inclination. If the optical axis of the lens is perpendicular to the plate surface, the fringes of equal inclination have the form of concentric rings with the center in the main focus of the lens.

This phenomenon is used to determine the refractive index of glass.

Interferential condition for the m-th minimum of reflected light according to Eq. (3.1.5) is

$$2hn\cos\beta_m = m\lambda.$$

For another minimum, k bands apart from the m-th, we have:

$$2hn\cos\beta_{m+k} = (m+k)\lambda. \tag{3.1.6}$$

After calculating the difference between Eqs (3.1.6) and (3.1.5), we get:

$$k\lambda = 2hn(\cos\beta_{m+k} - \cos\beta_m). \tag{3.1.7}$$

For small angles β_{m+k} and β_m, the functions $\cos\beta_{k+m}$ and $\cos\beta_k$ are expanded into a series and limited by the first approximation $\cos\beta \approx 1 - \frac{1}{2}\beta^2$:

$$|\cos\beta_{m+k} - \cos\beta_m| = \frac{\beta_{m+k}^2 - \beta_m^2}{2}. \tag{3.1.8}$$

According to the Snell's law and considering angles α_m and β_m small, we get:

$$n = \frac{\sin\alpha_m}{\sin\beta_m} = \frac{\alpha_m}{\beta_m}. \tag{3.1.9}$$

As can be readily seen from the right triangle $\triangle OBC$ in Fig. 3.1.ii, small angle α_m is:

$$\alpha_m = \frac{R_m}{2L}, \tag{3.1.10}$$

where R_m is the radius of the m-th dark ring; L is the distance between the glass plate and the screen.

Similar expression can be written for a $(m+k)$-th dark ring:

$$\alpha_{m+k} = \frac{R_{m+k}}{2L}. \tag{3.1.11}$$

Substituting Eqs (3.1.8) – (3.1.11) into the Eq. (3.1.7), we get:

$$n = \frac{h(R_{m+k}^2 - R_m^2)}{4kL^2\lambda}. \tag{3.1.12}$$

Thus, measuring the radii of two dark rings, thickness of the plate, and distance between the plate and the screen according to Eq. (3.1.12), we can calculate the index of refraction of the glass.

The great spatial and time coherence of laser emission allows us to use the laser light beam with the power of some milliwatts to observe interference fringes in the sufficiently thick plane-parallel plate.

Step-by-step Procedure of the Experiment

1. Turn on the power supply and the laser.
2. Place the screen with the lens near the laser so that the beam is aimed at the hole in the screen.
3. Adjust the position of the glass plate so that you clearly observe interference pattern (dark and bright rings) on the screen.
4. Measure radii of two dark rings R_m and R_{m+k} (it is advisable k is high).
5. Using the scale on the optics bench, measure the distance L between the screen and the front surface of the glass plate.
6. Calculate the refractive index of glass according to Eq. (3.1.12) if the laser wavelength $\lambda = 6328$ Å (1 Å$= 10^{-10}$ m), and the thickness of glass plate $h = 17$ mm.
7. Move the glass plate along the optics bench and repeat steps 3–6 again.

8. Find the average value of the refractive index of glass.

After-lab Questions

Case 1.

1. What is interference?
2. Derive the expression for the optical path difference Δ of rays reflected by the plane-parallel plate.
3. What are the fringes of equal inclination?
4. *Problem.* White light falls on the soap film at the angle $\alpha = 45°$. What its minimal thickness should be for a yellow-coloured ($\lambda = 600$ nm) film to be observed in reflected rays? Refractive index of soapsuds is $n = 1.33$.
   Answer: $h = 0.13\ \mu$m.

Case 2.

1. What is an the optical path difference of beams?
2. Derive the expression for calculating of the refractive index of glass using in this experiment.
3. Give the examples of the interference practical application in engineering.
4. *Problem.* A plane-parallel glass plate of the thickness $h = 1$ mm stands on the way of a monochromatic light with the wavelength $\lambda = 0.6\ \mu$m. Light falls on the plate normally. What angle φ is it necessary to turn the plate so that the optical distance changes by $\lambda/2$? Refractive index of glass is $n = 1.5$.
   Answer: $\varphi = 1.72°$.

Case 3.

1. What waves are called coherent?
2. What is an optical path?
3. Derive the conditions for constructive and destructive interference to occur.

4. *Problem.* Light propagates in the air. A glass plate of the thickness $h = 1$ mm stands in its way. How much will the optical path of the beam change by if the light falls on the plate: (a) normally; (b) at the angle $\alpha = 30°$.
   Answer: (a) increase by 0.5 mm; (b) increase by 0.548 mm.

LABORATORY EXPERIMENT 3.7

DETERMINATION OF A LIGHT WAVELENGTH WITH DIFFRACTION GRATING

Purpose of the Experiment: to determine wavelengths of two colours of visible light.

Equipment and Accessories: optics bench; lantern with two slits; diffraction grating; light filters.

Basic Methodology. Light from two slits falls normally on a diffraction grating mounted behind a movable plate with light filters. As the slits are situated at different heights, so their diffraction patterns are seen one above another, and the same order diffraction maxima from the slits can be matched. Wavelengths are computed by calculating diffraction angles and using the condition for diffraction maxima.

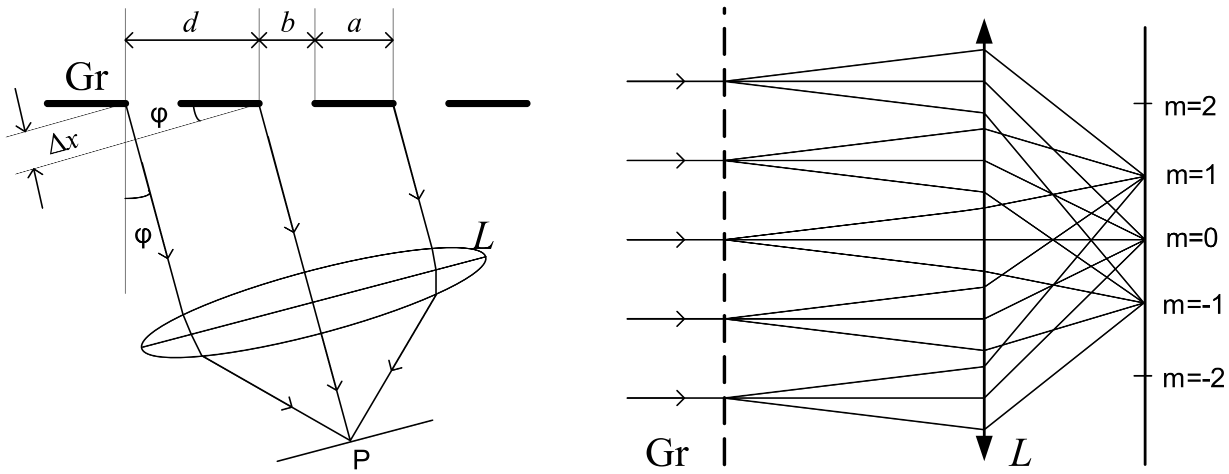
Recommended Pre-lab Reading: [1] 36.1, 36.2, 36.5; [2] 38.1, 38.2, 38.3; [3] 38.1, 38.2, 38.4.

Pre-lab Questions

1. What is a diffraction grating?
2. Draw the pattern diagram of the diffraction grating.
3. What condition must be met for a maximum to occur in the diffraction pattern for a diffraction grating?

Theoretical Introduction

The diffraction grating, a useful device for analyzing light sources, consists of a large number of equally spaced parallel slits. A transmission grating can be made by cutting parallel grooves on a glass plate with a precision ruling machine. The spaces between the grooves are transparent to the light and hence act as separate slits. A reflection grating can be made by cutting parallel grooves on the surface of a reflective material. The reflection of light from the spaces between the grooves is specular, and the reflection from the grooves cut into the material is diffuse. Thus, the spaces between the grooves act as parallel sources of reflected light, like the slits in a transmission grating.



a *b*

Figure 3.7.i

In Fig. 3.7.i, you can see cross sections of a transmission grating. The slits are perpendicular to the plane of the page, and an interference pattern is formed by the light that is transmitted through the slits. All slits have the same width b (see Fig. 3.7.i.a) and equal distances a between adjacent ones. The spacing d between centers of the adjacent slits, or $d = a + b$, is called the grating spacing.

In Fig. 3.7.i,b, a plane wave is incident from the left, normal to the plane of the grating. The pattern observed on the screen (we need to use a convex lens to gather parallel rays at one point) is the result of the combined effects of interference and diffraction. Each slit produces diffraction, and the diffracted beams interfere with one another to produce the final pattern.

The waves from all slits are in phase as they leave the slits. However, for some arbitrary direction φ measured from the normal to the grating plane, the waves must travel different path lengths before reaching the screen. From Fig. 3.7.i.a, note that the path difference Δ between rays from any two adjacent slits is equal to $d \sin \varphi$. If this path difference equals one wavelength or some integer multiple of a wavelength, then waves from all slits are in phase at the screen and a bright fringe is observed. Therefore, the condition for maxima in the interference pattern at the angle φ is

$$d \sin \varphi = m\lambda \quad m = 0, \pm 1, \pm 2 \ldots \tag{3.7.1}$$

Such maxima are called principal. Between two principal maxima, there are $(N-1)$ minima (N is a number of slits). Between two such consecutive minima, the intensity has to have a maximum; these maxima are known as secondary maxima. These are of much smaller intensity than principal maxima, therefore, secondary maxima are perceived as a weak background.

We can use Eq. (3.7.1) to calculate the wavelength if we know the grating spacing d and the angle φ. If the incident radiation contains several wavelengths, the mth-order maximum for each wavelength occurs at a specific angle.

In Fig. 3.7.ii,a, there is a scheme for observing interference pattern in this experiment.

Light beams from the slits S_1 and S_2, passing through the diffraction grating, produce two diffraction spectra displaced relative to each other (Fig. 3.7.ii,b).

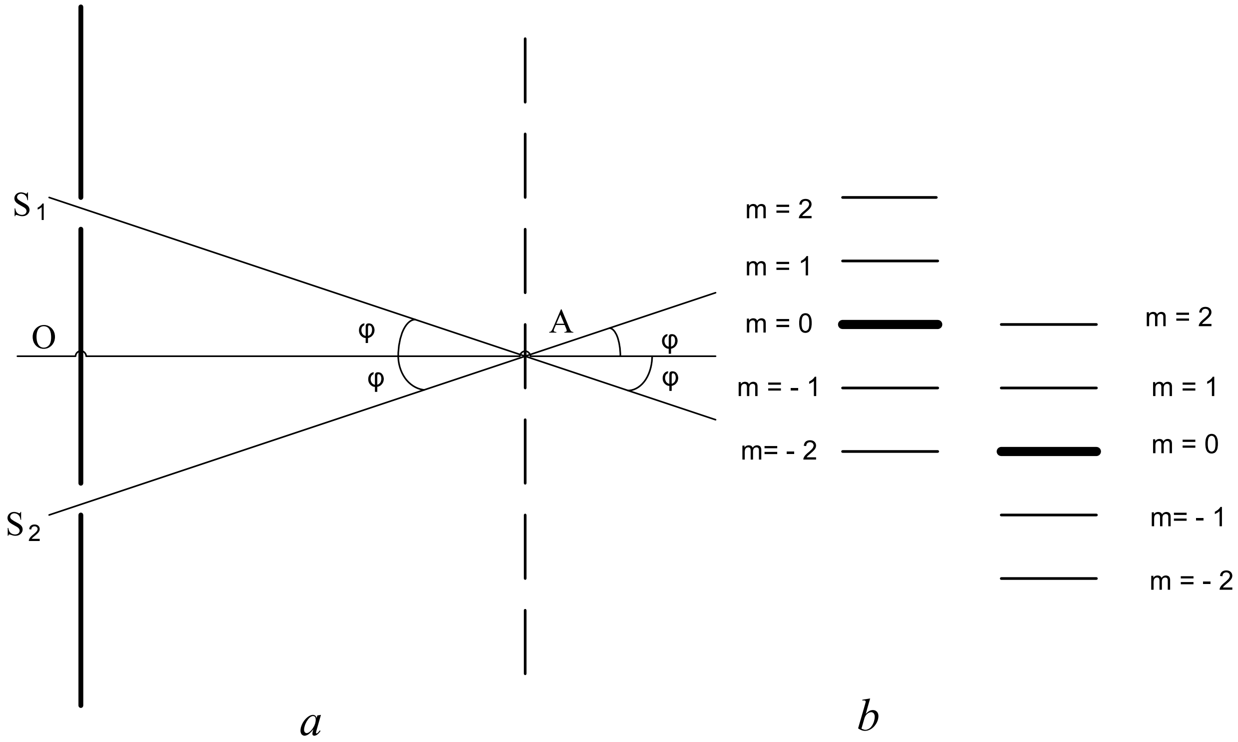


Figure 3.7.ii

Moving the diffraction grating along the optics bench, one can receive the overlapping of the slit S_1 left diffraction image with the slit S_2 right diffraction image. If you overlap the first–order maximum ($m = 1$) from one slit with the maximum of $m = -1$ order from another (Fig. 3.7.ii,b), the position of slits overlapped images in your eye is exactly on the optics bench axes. Virtual images of overlapped maxima you see midway between the slits. As you can see in Fig. 3.7.ii,a, OS_1A is a right triangle, so

$$\tan\varphi = \frac{OS_1}{AO} = \frac{S_1S_2}{2AO}. \tag{3.7.2}$$

From $\tan\varphi$, you can find out the magnitude of $\sin\varphi$:

$$\sin\varphi = \frac{\tan\varphi}{\sqrt{1+\tan^2\varphi}}.$$

The above is correct for overlapping the second- and third-order maxima ($m = \pm 2, \pm 3$). So, the expression for a wavelength calculation is

$$\lambda = \frac{d\sin\varphi}{m}. \tag{3.7.3}$$

Description of the Equipment

The laboratory facility for this experiment consists of an optics bench, a lantern with two slits, a diffraction grating, and light filters arranged on the bench. Slits are equidistant to the left and to the right from the optics bench. For ease of observations, they are at different height, and the bottom of the upper slit is at the same level as the top of the lower one. The holder of light filters and diffraction grating can be moved along the optic bench.

Step-by-step Procedure of the Experiment

1. Turn on the lantern and place the first light filter in front of the diffraction grating.
2. Looking through and above the diffraction grating, find two slit images corresponding to the straight rays, i.e. the 0-th order spectra. These images are the brightest ones.
3. Moving the holder of diffraction grating, find the position where slit virtual images of the first order ($m = \pm 1$) are situated one above another.
4. Measure the distance from the lantern to the holder. It is the distance AO on the scheme in Fig. 3.7.ii,a. (The distance S_1S_2, between the centers of the slits and the grating spacing d are given.) From Eq. (3.7.2), calculate $\tan\varphi$ and find out $\sin\varphi$. Determine the wavelength λ_1 for $m = 1$ from Eq. (3.7.3).
5. Repeat the steps 3 and 4 for the second-order spectra ($m = \pm 2$) and calculate the wavelength λ_2 for it. If it's possible, repeat the same for the third-order spectra ($m = \pm 3$) and calculate λ_3.
6. Calculate the average value of the wavelength λ_{av}.
7. Repeat the steps 3 trough 6 for another light filter.

8. Remove the light filter. Sketch the result of visible light diffraction (0-th, 1-st and 2-nd order maxima) to scale and mark the colors.

Warning. Don't touch the surface of the diffraction grating.

After-lab Questions

Case 1.

1. What is diffraction? What types of diffraction do you know?
2. Derive the condition for diffraction minima at diffraction from one slit.
3. Draw a sketch of the diffraction pattern when visible light passes through a diffraction grating and explain it.
4. *Problem.* Monochromatic light ($\lambda = 600$ nm) is at normal incidence on a diffraction grating that has 200 slits/mm. What greatest order of maximum can be observed from this diffraction grating?
   Answer: $m = 8$.

Case 2.

1. Formulate the Huygens-Fresnel principle.
2. Derive the condition for principal maxima at diffraction from diffraction grating.
3. Draw a sketch of the facility optic scheme for diffraction spectra observation. What performs a function of a convex lens?
4. *Problem.* If a diffraction grating produces its fifth-order bright band at an angle of 18° for light of wavelength 600 nm, find the number of slits per millimeter for the grating.
   Answer: 103 slits/mm.

Case 3.

1. Describe the method of Fresnel zones.
2. Can a diffraction grating for visible light be used for X–rays diffraction? Why?
3. Explain the presence of secondary maxima in the pattern from a diffraction grating.

4. *Problem.* What greatest order of spectrum can be observed with the diffraction grating you have used in your experiment ($d = 0.01$ mm) for the wavelength $\lambda = 600$ nm?
   Answer: m=16.

LABORATORY EXPERIMENT 3.8

INVESTIGATION OF LIGHT POLARIZATION

Purpose of the Experiment: to study the polarization of light, to verify Malus law, to find the Brewster angle for glass and its index of refraction.

Equipment and Accessories: optics bench; light source connected to the rectifier; rectifier networked with voltage 220 V (allows users to adjust light source voltage and change its brightness); two polaroids (each enclosed in the mount with a dial for measuring of a polaroid rotation angle around the horizontal axis); black mirror in the mount that allows users to rotate it around the vertical axis; photocell on the tripod attached to the microammeter.

Basic Methodology. Magnitude of a photocell current depends on the intensity of light falling on it. Therefore, measuring current with microammeter, we find out intensity change of natural light which has passed through two polaroids.

Recommended Pre-lab Reading: [1] 33.5; [2] 34.5; [3] 38.6.

Pre-lab Questions

1. Draw a schematic diagram of an electromagnetic wave propagating at velocity c in the positive x direction.
2. What can you say about the direction in which the electric field is vibrating in a beam of light emitted by the ordinary light source?
3. What light is said to be plane-polarized?

Theoretical Introduction

There is a number of ways an unpolarized light can be converted into a plane-polarized light.

Polarization by Selective Absorption

The most common technique for producing polarized light is to use a material that transmits waves which electric fields vibrate in a plane parallel to a certain direction and that absorbs waves which electric fields vibrate in all other directions.

In 1938, E. H. Land (1909–1991) discovered a material, which he called polaroid, that polarizes light through selective absorption by oriented molecules. This material is fabricated in thin sheets of long-chain hydrocarbons. The sheets are stretched during manufacture so that the long-chain molecules align. After a sheet is dipped into a solution containing iodine, the molecules become good electrical conductors. However, conduction takes place primarily along the hydrocarbon chains because electrons can move easily only along the chains. As a result, the molecules readily absorb light which electric field vector is parallel to their length and allow light through which electric field vector is perpendicular to their length.

It is common to refer to the direction perpendicular to the molecular chains as the polarizing, or transmission, axis. In an ideal polarizer, all light with $\vec{\boldsymbol{E}}$ parallel to the polarizing axis is transmitted, and all light with $\vec{\boldsymbol{E}}$ perpendicular to the polarizing axis is absorbed.

Let us consider an unpolarized light beam incident on a first polarizing sheet, called the polarizer. If the polarizing axis is oriented vertically the light transmitted through this sheet is polarized vertically. A second polarizing sheet, called the analyzer, intercepts the beam. The analyzer polarizing axis is set at an angle α to the polarizer axis. We call the electric field vector of the first

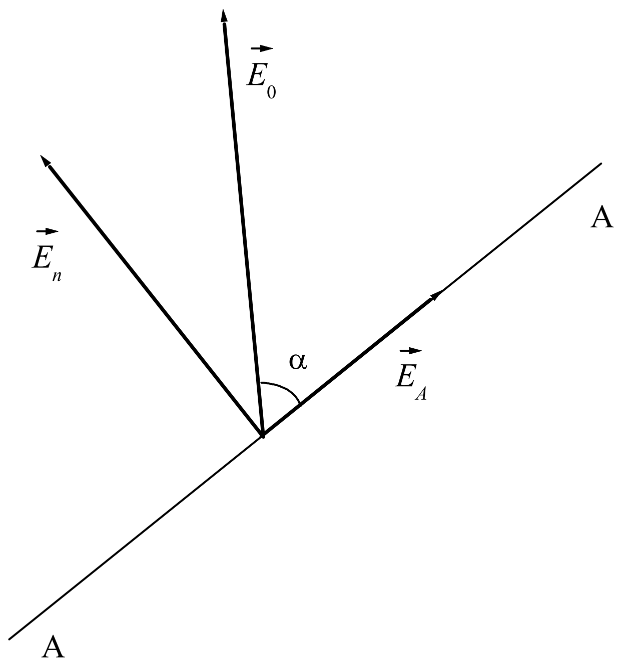


Figure 3.8.i

transmitted beam E_0. The component of E_0 perpendicular to the analyzer axis is completely absorbed. The component of E_0 parallel to the analyzer axis, which is allowed through by the analyzer, is $E_0 \cos\alpha$ (See Fig. 3.8.i). Because the intensity of the transmitted beam varies as the square of its magnitude, we conclude that the intensity of the (polarized) beam transmitted through the analyzer varies as

$$I = I_0 \cos^2 \alpha, \tag{3.8.1}$$

where I_0 is the intensity of the polarized beam incident on the analyzer. This expression, known as Malus's law, applies only if the incident light passing through the analyzer is already linearly polarized.

We should take into consideration that polaroids are not ideal polarizers. They do not provide complete polarization of light and are unequally transparent for rays of different colours. For this reason, the intensity of light transmitted through the analyzer consists of intensities of two beams, polarized and unpolarized ones. The intensity I_1 of the unpolarized beam does not depend

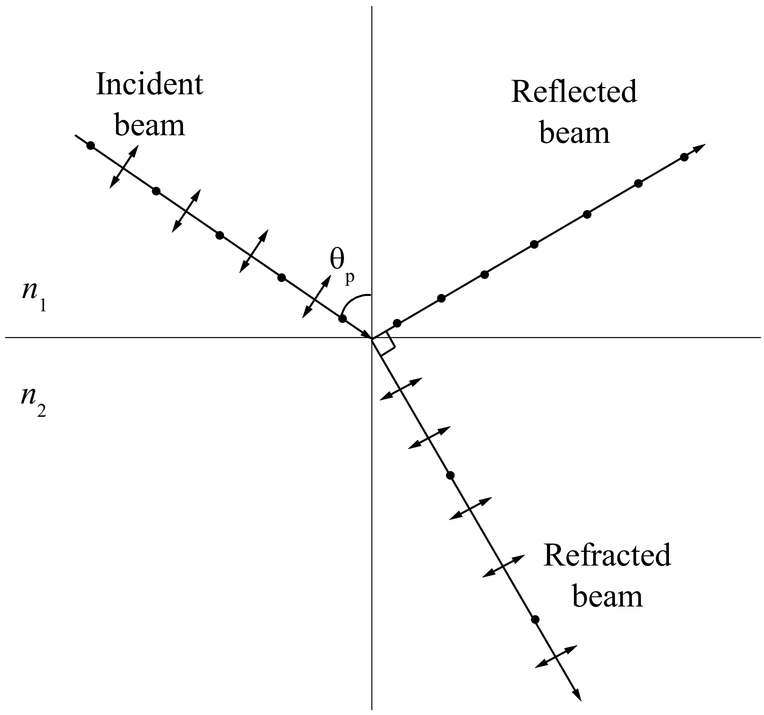


Figure 3.8.ii

on the angle α between the polarizing axes of the polarizer and the analyzer. Thus, for polaroids, the intensity of light transmitted through the analyzer is determined by the relation:

$$I = I_1 + I_0 \cos^2 \alpha. \tag{3.8.2}$$

Polarization by Reflection

When an unpolarized light beam is reflected from a surface, the reflected light may be completely polarized, partially polarized, or unpolarized, depending on the angle of incidence. If the angle of incidence is 0°, the reflected beam is unpolarized. For other angles of incidence, the reflected light is polarized to some extent, and for one particular angle of incidence, the reflected light is completely polarized. Let us now investigate reflection at that special angle.

Suppose that an unpolarized light beam is incident on a surface (as an incident beam in Fig. 3.8.ii). Each individual electric field vector can be resolved into two components: one parallel to the surface (and perpendicular to the page in Fig. 3.8.ii, represented by the dots), and the other (represented by the arrows) perpendicular both to the first component and to the direction of propagation.

Thus, the polarization of the entire beam can be described by two electric field components in these directions. It is found that the parallel component reflects more strongly than the perpendicular component, and this results in a partially polarized reflected beam. Furthermore, the refracted beam is also partially polarized.

Now suppose that the angle of incidence is varied until the angle between the reflected and refracted beams is 90°, as in Fig. 3.8.ii. At this particular angle of incidence, the reflected beam is completely polarized (with its electric field vector parallel to the surface), and the refracted beam is still only partially polarized. The angle of incidence at which this polarization occurs is called the polarizing angle θ_p, which satisfies the equation

$$n = \tan \theta_p, \tag{3.8.3}$$

where n is the relative index of refraction. This expression is called Brewster's law, and the polarizing angle θ_p is sometimes called Brewster's angle, after its discoverer, David Brewster (1781–1868). Because n varies with wavelength for a given substance, Brewster's angle is also a function of wavelength.

Description of the Equipment

The picture of laboratory facility is represented in Fig. 3.8.iii where on the optic bench from left to right, there are a light source (connected to the power supply), a polarizer, an analyzer, and a photocell as a light receiver connected to a sensitive microammeter for photocurrent recording.

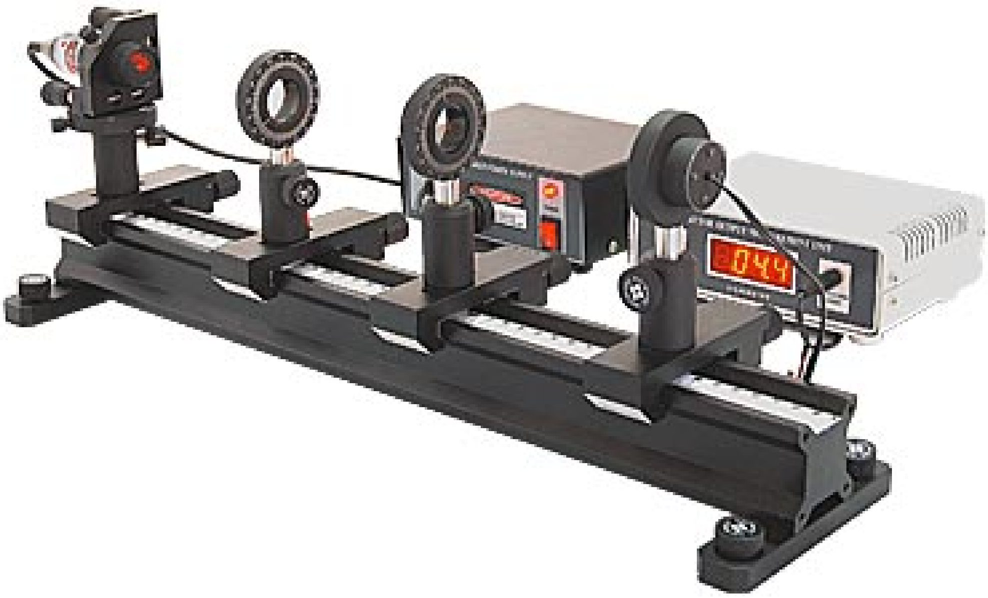


Figure 3.8.iii

Step-by-step Procedure of the Experiment

Task 1. Study of Malus's law.

1. Arrange devices on the optic bench, as shown in Fig. 3.8.iii.
2. Turn on the power supply of the light source.
3. Turn on the power supply with microammeter connected to the photocell.
4. Set up the light source, the polaroids, and the photocell so that the beam passes through both the polarizers and falls on to the photocell.
5. Adjust the light source voltage (use a handle on the power supply) and limit of effective range of the microammeter (number of keys on it) so that a microammeter measuring cursor does not transcend the scale. The distance between the light source and the photocell should be not less than 1 m.
6. Cursor on the mount of the analyzer set to zero on the scale of the analyzer.
7. Rotating the polarizer, assign a maximum value of intensity of the light transmitted through both the polarizers. It corresponds to the maximum value of photocurrent recorded by

the microammeter.

Remember! For accurate measurements, the maximum value of photocurrent should be within the last quarter of the microammeter scale. So, if it's necessary adjust the intensity of light source again.

8. Rotate the analyzer over the range 0° to 180° and every 10° measure the photocurrent. Take data down into the table.
9. Construct a graph of the photocurrent I versus $\cos^2\alpha$.
10. Calculate the degree of polarization of light passing through the analyzer. The degree of light polarization is determined by the expression
$$P = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}},$$
where $I_{\max}$ and $I_{\min}$ are the maximum and minimum values of light intensity corresponding to the two orthogonally related directions of light vibrations in the beam.

Task 2. Determination of the refractive index of glass by its Brewster's angle.

1. Remove the analyzer and place a black mirror instead.
2. Set the mirror surface perpendicular to the incident beam. Set the pointer on the limb on the horizontal mount of the mirror to a handy mark for readout (e.g., 0 or 180°).
3. Turn the mirror at some angle ($\sim 40\ldots 50°$) and, rotating the polarizer, visually observe the light source image given by the rays reflected from the mirror. Fix the polarizer at the position at which the intensity of the reflected beam is minimal.
4. Rotating the mirror, receive less intensity of the source image. Continue to rotate the polarizer and the mirror by turns till the intensity of the reflected beam becomes minimal. If polarized light falls on the mirror, there is no reflected beam under two conditions:

(a) in the incident light, vector $\vec{\boldsymbol{E}}$ vibrates in the plane of incidence;
(b) the angle of incidence equals the Brewster's angle.

As a polaroid is used as a polarizer in this work, you can receive only minimal intensity of the reflected beam but not its absence.

5. Mark the position of the pointer on the limb. Difference between the final and the initial values of the pointer position is equal to the Brewster's angle θ_p.
6. Using Eq. (3.8.3) compute the refractive index of glass, which a black mirror is made of.

After-lab Questions

Case 1.

1. What light is called natural, plane-polarized, partially polarized?
2. Describe the phenomenon of double refraction (or birefringence). Enumerate properties of ordinary and extraordinary rays.
3. Prove that the intensity of light passing through any ideal polarizer is equal to half of intensity of the incident natural light (under absence of absorption in the material of the polarizer).
4. *Problem.* What is the refractive index of glass if the reflected beam is completely polarized? The angle of refraction is 30°. *Answer:* $n = 1.73$.

Case 2.

1. Light falls on the surface of the dielectric. How are reflected and refracted beams polarized?
2. Formulate and derive the formula for Malus's law.
3. Explain the operating principle of polaroid.

4. *Problem.* Natural light of intensity I_0 passes through the system of two crossed polarizers. What is the intensity of light transmitted through the system if the third polarizer is placed between the two, and its polarizing axis is set at an angle α to the first polarizer axis?
   Answer: $I = \dfrac{I_0}{8}\sin^2 2\alpha$.

Case 3.

1. Draw a diagram for the situation when light strikes a surface at the polarizing angle. Using it and the law of refraction, derive the expression for Brewster's law.
2. Explain the operating principle of Nicol prism?
3. What is a degree of polarization? For what type of polarized light can this notion be used?
4. *Problem.* What is the angle between the polarizing axes of the polarizer and the analyzer, if the intensity of natural light passed through them decreases four times? Light absorption should be neglected.
   Answer: $\alpha = 45°$.

LABORATORY EXPERIMENT 3.12

STEFAN-BOLTZMANN CONSTANT DETERMINATION

Purpose of the Experiment: to learn an operating principle of an optical pyrometer, to determine the Stefan–Boltzmann constant.

Equipment and Accessories: disappearing filament pyrometer LOP-72; pyrometer power supply; universal digital voltmeter B7-22A; ammeter; voltmeter; autotransformer.

Basic Methodology. Using a disappearing filament pyrometer, the temperature of a heated body is determined. Radiated power of a heated body is calculated by reading of an ammeter and a voltmeter. The Stefan–Boltzmann constant is calculated by the Stefan-Boltzmann law.

Recommended Pre-lab Reading: [1] 17.7, 38.8; [2] 17.5, 40.1; [3] 20.7, 40.1.

Pre-lab Questions

1. What is thermal radiation?
2. What properties does it have?
3. What assumption was made by Planck to explain physical laws of thermal radiation?

Theoretical Introduction

All matter, at any temperature, absorbs and emits electromagnetic radiation, and across the full range of frequencies. The basic physical emission mechanism is that atoms go into excited states when the temperature of the matter they comprise is raised. They radiate energy when returning to their normal states.

An ideal surface that absorbs all wavelengths of electromagnetic radiation incident upon it is also the best possible emitter of electromagnetic radiation at any wavelength. Such an ideal surface is called a **blackbody**, and the continuous- spectrum radiation that it emits is called **blackbody radiation**. By 1900, this radiation had been studied extensively, and the following had been established.

The total intensity R^* (the average rate of energy radiation per unit surface area or average power per area) emitted from the surface of an ideal radiator is proportional to the fourth power of the absolute temperature. This relationship is called the Stefan-Boltzmann law:

$$R^*(T) = \sigma T^4, \qquad (3.12.1)$$

where σ is a fundamental physical constant called the Stefan-Boltzmann constant. In SI units, $\sigma = 5.670400 \cdot 10^{-8}$ W/(m^{2}K^4).

The total intensity R emitted from a real surface is expressed as

$$R(T) = e\sigma T^4,$$

or in terms of power P radiated from the surface A of the object ($R = P/A$):

$$P = Ae\sigma T^4, \tag{3.12.2}$$

where e is the **emissivity**, and T is the surface temperature in kelvins.

The value of e can vary between zero and unity, depending on the properties of the object surface. The emissivity is equal to the **absorptivity** which is the fraction of the incoming radiation that the surface absorbs.

If the object is a spiral of an incandescent lamp, then its power equals

$$P = IU, \tag{3.12.3}$$

where I is a current in the spiral and U is its voltage.

Substituting Eq. (3.12.3) in Eq. (3.12.2), we receive the expression for experimental determination of the Stefan-Boltzmann constant:

$$\sigma = \frac{IU}{AeT^4}. \tag{3.12.4}$$

To determine temperature of a body based on thermal radiation laws, pyrometers are used. First, using a pyrometer, a brightness temperature is received, and then from the graph of a real temperature versus brightness temperature, the real one is determined.

Description of the Equipment

A sketch of the disappearing filament pyrometer LOP-72 is shown in Fig. 3.12.i.

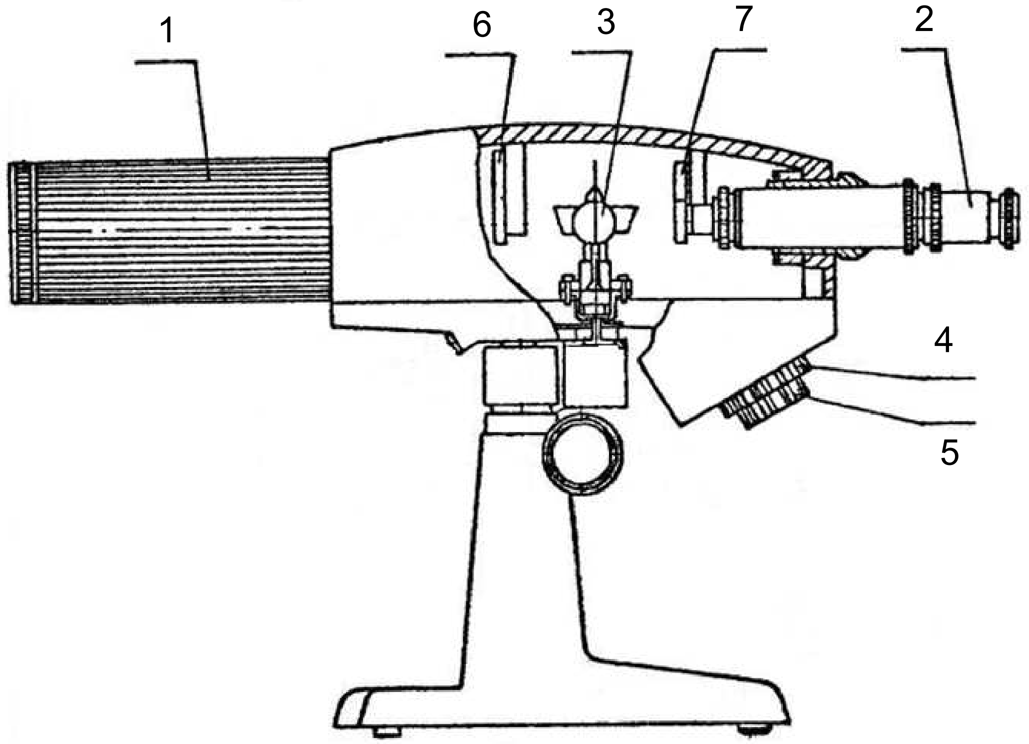


Figure 3.12.i

Here, 1 is an objective, 2 is an ocular, 3 is a pyrometric lamp with the arc-shaped filament; 4, 5 are rheostats for rough and precise adjustment of a filament heating; 6 is an absorption screen, and 7 is a light filter.

The scheme of the experimental setup is shown in Fig. 3.12.ii.

A studied object is a tungsten spiral of the incandescent lamp 1. An electric circuit for the tungsten spiral heating consists of an autotransformer 2 connected to the electrical network with voltage $U = 220$ V, voltmeter 3, and ammeter 4. The electric circuit of a pyrometer 5 includes: pyrometric lamp 6, stabilized power supply of the pyrometric lamp 7, milliammeter 8 for measuring of a filament current in the pyrometric lamp (the universal digital voltmeter B7-22A is used as a milliammeter).

Step-by-step Procedure of the Experiment

1. Place the pyrometer before the object which temperature is

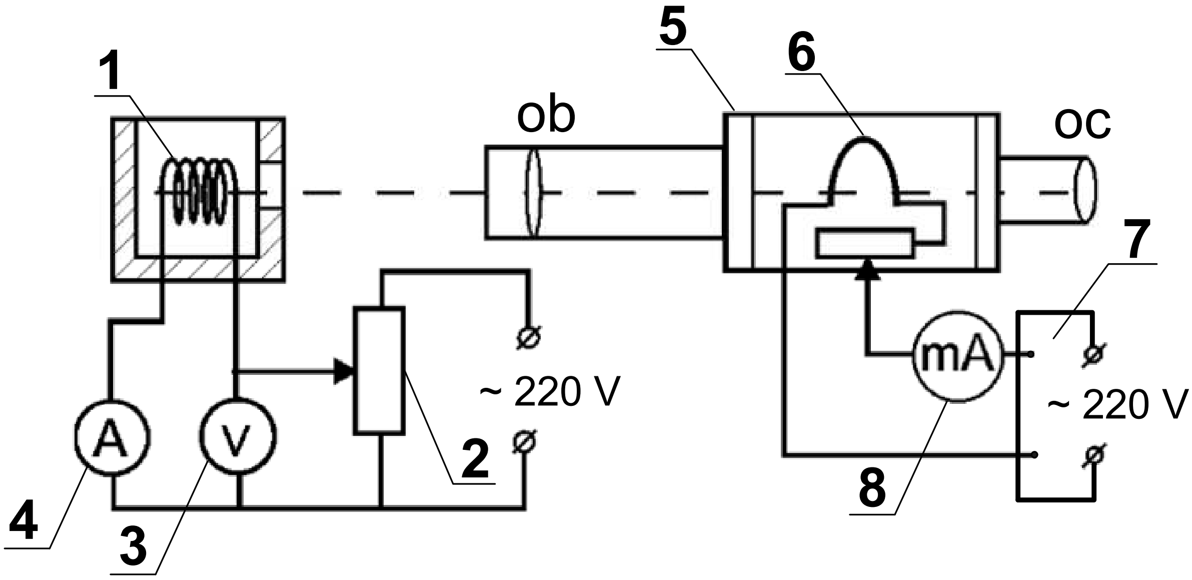


Figure 3.12.ii

measured at the distance about 0.6 m.

2. Check the rheostats' knobs 3 and 4 (see Fig. 3.12.ii) to be at the leftmost position.
3. Turn on the pyrometer power supply and the digital voltmeter.
4. Increasing current with the rheostat 4, achieve a visible glow of the pyrometric lamp filament, and with ocular 2, try to obtain its sharp image.
5. Switch the autotransformer 2 on (see Fig. 3.12.i) and turning its handle, make the incandescent lamp spiral (an object of studying) white-hot. Rotating the tube of the objective 1 (see Fig. 3.12.ii), try to get a sharp image of the spiral. Meanwhile, the images of incandescent lamp spiral and pyrometric lamp filament, observing with the ocular 2, should be equally sharp.
6. On the pyrometer, place a light filter handle to the position 1 and an absorption screen handle to the position 1.
7. Obtain the same brightness for the pyrometric lamp filament and the tungsten spiral, i.e. disappearing of the filament against the lamp spiral (an object of studying), with rheostats 3 and 4.

Table 3.12.i

#	I, mA	T_b, °C	T, °C	T, 10^3 K	T^4, 10^{12} K^4	U, V	I, A	A, m^2	e	σ, W/(m^{2}K^4)
1										
2										

8. With digital voltmeter, measure a current I_f going through the pyrometric lamp filament.

   Warning. Current in the pyrometric lamp filament should not exceed 460 mA.

   Record this and further measurements in Table 3.12.i.

9. With the graph $I_f = F(T_b)$ which is attached, determine a brightness temperature T_b of a tungsten spiral of the incandescent lamp.

10. With the help of the following graph $T = f(T_b)$ determine a real temperature T of the tungsten spiral.

    Convert it from Celsius degrees to Kelvins. To ease further calculation, represent the temperature T as $T = 1.??? \cdot 10^3$ K.

11. Measure a current in the tungsten spiral I and its voltage U with ammeter 4 and voltmeter 3 (see Fig. 3.12.i) .

12. Determine the Stefan-Boltzmann constant σ from Eq. (3.12.4). (The emitting area of the tungsten spiral A and its emissivity e are given on the diagrams).

13. Change brightness of the incandescent lamp and repeat steps 7–12.

14. Determine an average value of the Stefan-Boltzmann constant.

After-lab Questions

Case 1.

1. Formulate the definitions of the total intensity and the spectral emittance. How do they relate to each other?
2. Draw the dependence graph of the thermal radiation energy density on the temperature.
3. Formulate the Kirchhoff law for thermal radiation.
4. *Problem.* A 100-W incandescent light bulb has a cylindrical tungsten filament 30.0 cm long, 0.40 mm in diameter, and with an emissivity of 0.26. (a) What is the temperature of the filament? (b) For what wavelength does the spectral emittance of the bulb peak?
   Answer: 2060 K, 1.41 μm.

Case 2.

1. Formulate the Stefan-Boltzmann law.
2. Why is the temperature of a real body, determined in your experiment with pyrometer, called a brightness temperature?
3. What body is called a blackbody? Do they exist really?
4. *Problem.* The shortest visible wavelength is about 400 nm. What is the temperature of an ideal radiator whose spectral emittance peaks at this wavelength?
   Answer: 7250 K.

Case 3.

1. Formulate the Wien displacement law.
2. Show that for large values of λ the Planck formula agrees with the Rayleigh formula.
3. Describe a model of a blackbody. Why can it be considered in this way?
4. *Problem.* Two stars, both of which behave like ideal blackbodies, radiate the same total energy per second. The cooler one

has a surface temperature T and 3.0 times the diameter of the hotter star. What is the temperature of the hotter star in terms of T?
Answer: $\sqrt{3}T$.

LABORATORY EXPERIMENT 3.13
STUDY OF THE PHOTOELECTRIC EFFECT

Purpose of the Experiment: to study the dependences of a photocurrent on photocathode illuminance and voltage between the photocathode and the anode.

Equipment and Accessories: cesium-antimonide photocell, microammeter, voltmeter, rheostat, rectifier, incandescent lamp.

Basic Methodology. A photocell is illuminated by the light of an incandescent lamp, and due to the photoelectric effect, photocurrent occurs. Using a proper circuit, and varying either illuminance of the photocathode or the voltage at the photocell, the photocurrent is measured.

Recommended Pre-lab Reading: [1] 38.1, 38.2; [2] 40.1; [3] 40.1.

Pre-lab Questions

1. Metals contain free electrons. Can they leak out of the metal freely? Why?
2. What effect is called a photoelectric one?
3. What nature, wave or quantum, does radiation exhibit in the photoelectric effect?

Theoretical Introduction

At the end of the XIXth century, V. Galavax's, A. Rigi's (1888) and A. Stoletov's (1888–1890) experiments showed that under the light influence a metallic cathode emits negative charges. The effect of electrons emission from rigid and liquid substances when light strikes a surface is called photoelectric effect. Schematic diagram of the Stoletov's experiment is shown in Fig. 3.13.i.

Light from the source S falling on the metal cathode C causes a current of emitted electrons. The current from cathode to anode A (which has the shape of a grid) is measured by the galvanometer (G). The photocurrent dependence on the voltage between the anode and the cathode is shown in Fig. 3.13.ii and called volt-ampere characteristic (illuminance $E = \text{const}$).

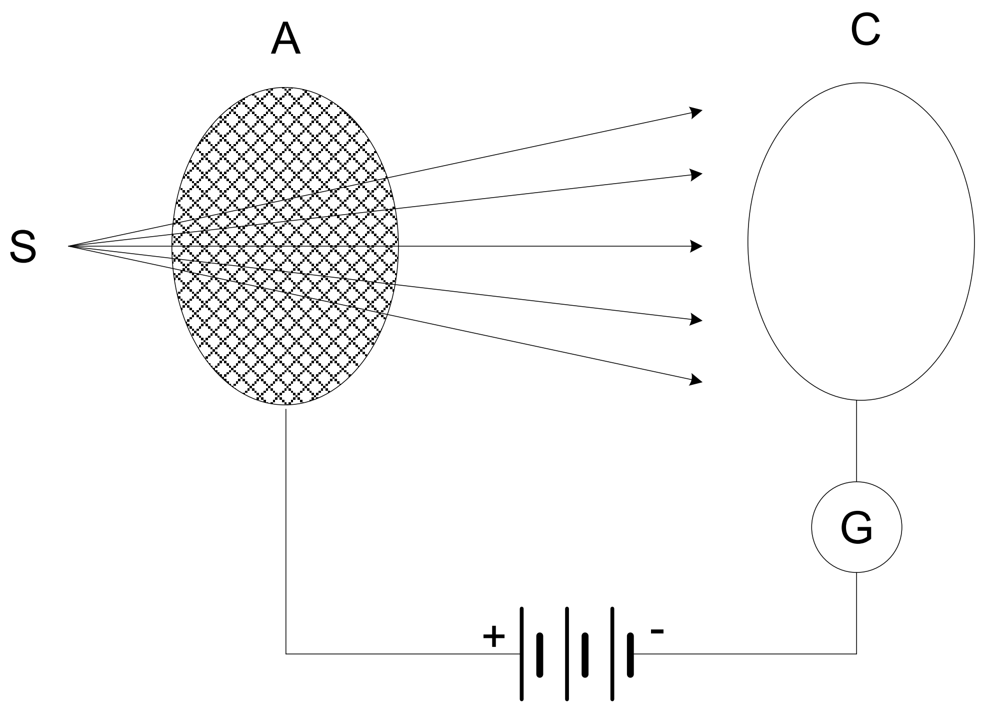


Figure 3.13.i

Experimentally, the following laws of photoelectric effect were obtained:

- the maximum speed of photoelectrons depends on the frequency of light and doesn't depend on the intensity of light;
- for each material, no photoelectrons at all are emitted unless the frequency of the light is greater than some minimum value

called the *threshold frequency*. This minimum frequency f_0 depends on the material of the cathode and its surface condition;

- the number of electrons n, emitted from the cathode per unit of time is proportional to the intensity of light, and photocurrent of saturation $I_s = n\,e$ (see Fig. 3.13.ii).

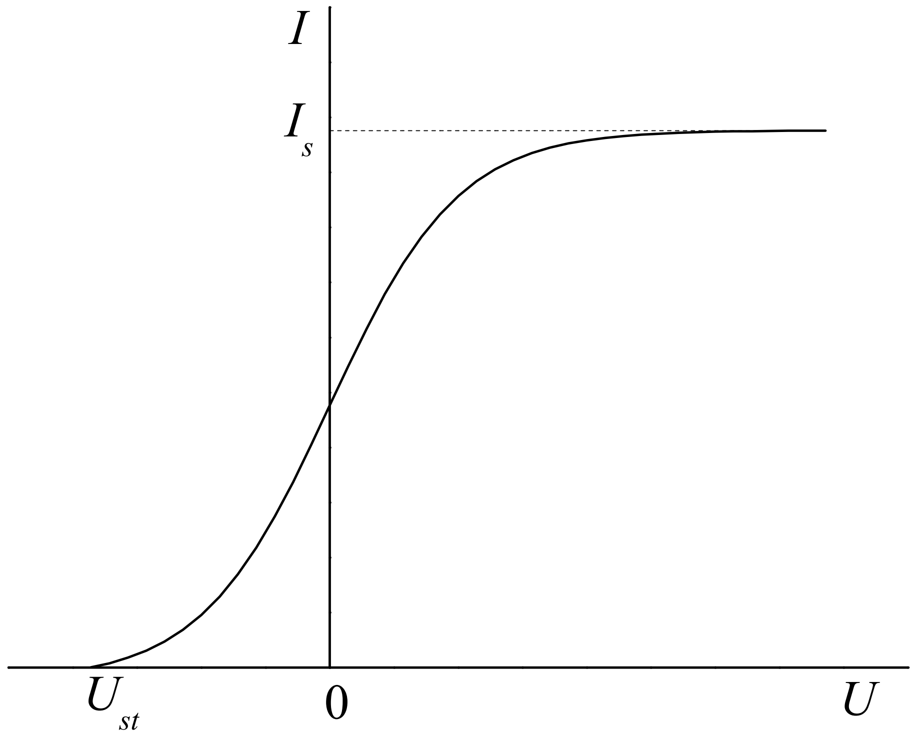


Figure 3.13.ii

These laws can't be understood without involving quantum ideas. Einstein explained these phenomena by postulating that electrons are emitted because electrons absorb individual photons. The photons that correspond to radiation of frequency f carry energy $E_f = hf$. If there is a minimum energy ϕ required to liberate an electron, then no electrons will be emitted when hf is less than ϕ. When hf exceeds ϕ, the excess energy can go into kinetic energy of the emitted electrons. Thus, applying the conservation energy law Einstein received:

$$hf = \phi + \frac{mv_{\max}^2}{2}, \qquad (3.13.1)$$

where h is the Plank's constant, ϕ is the *work function* (depends on the material of the cathode and its surface condition), $v_{\max}$ is the maximum velocity which an electron is able to have, and m is the mass of an electron.

We can determine the maximum kinetic energy of emitted electrons by making the potential of the anode relative to the cathode, V_{AC}, just negative enough so that the current stops. This occurs at $V_{AC} = -U_{st}$ where U_{st} is called the *stopping potential* (see Fig. 3.13.ii). As an electron moves from the cathode to the anode, negative work $-eU_{st}$ – is done on the (negatively charged) electron; the most energetic electron leaves the cathode with kinetic energy $K_{\max} = mv_{\max}^2/2$ and has a zero kinetic energy at the anode. So, we have

$$\frac{mv_{\max}^2}{2} = eU_{st}. \tag{3.13.2}$$

Hence, by measuring the stopping potential, we can determine the maximum kinetic energy with which electrons leave the cathode. (We ignore any effects due to differences in the materials of the cathode and anode.)

Greater intensity of falling radiation at a particular frequency means a proportionally greater number of photons per second absorbed, and thus a proportionally greater number of electrons per second emitted as well as the proportionally greater current.

Making an experiment, it's handier to measure the dependence of photocurrent on the illuminance of the photocathode which is proportional to the intensity of falling radiation.

Illuminance E of a point source of light is directly proportional to the luminous intensity I_l of the source and inversely proportional to the squared distance r between the source and the photocathode

$$E = \frac{I_l}{r^2}. \tag{3.13.3}$$

Therefore, if the source is moved relative to the photocell, its illuminance changes according to Eq. (3.13.3) and hence the photocurrent I changes inversely proportional to the squared distance r between the source and the photocell:

$$I = f\left(\frac{1}{r^2}\right). \tag{3.13.4}$$

Description of the Equipment

To research the laws of photoelectric effect, a vacuum cesium-antimonide photocell with the central anode is used. The photocell is a vacuum glass spherical vessel. A half of its internal surface is covered by layers of antimony and cesium, and as a result, a compound CsSb is formed which is used as a photocathode.

The photocell protected from the day light is placed on the optical bench in front of the incandescent lamp which is considered to be a point light source. The photocell is connected to the electrical network through a rectifier. Photocell circuit consists of a rheostat for changing the voltage on the photocell, a voltmeter for measuring the voltage, and a microammeter for measuring the photocurrent.

Step-by-step Procedure of the Experiment

Task 1. Dependence of the photocurrent on the illuminance of the photocell.

1. Place the lamp and the photocell at the same heights. Move the lamp to the photocell as close as possible ($r = 15\ldots 20$ cm).
2. Turn on the rectifier and give the voltage on the photocell $U = 150$ V.
3. Turn on the power of the lamp.
4. While increasing the distance r by 5 cm, take readings of photocurrent I versus r.
5. For every r (m), calculate $\dfrac{1}{r^2}$.

6. Plot a graph of photocurrent I versus $\frac{1}{r^2}$, i. e. $I = f(E)$.

Task 2. Volt-ampere characteristics of photocell.

1. Place the lamp at the distance 25 cm from the photocell.
2. While increasing the voltage from 0 to 150 V by 10 V, take readings of photocurrent I versus U.
3. Place the lamp at the distance 35 cm from the photocell (the illuminance of the photocathode has changed), and repeat step 2.
4. Place the lamp at the distance 45 cm from the photocell (the illuminance of the photocathode has changed again), and repeat step 2.
5. Plot graphs of volt-ampere characteristics, i. e. $I = f(U)$, of photocell for different illuminances at the same diagram.

After-lab Questions

Case 1.

1. Formulate the laws of photoelectric effect.
2. How can they be explained?
3. What is the work function?
4. *Problem.* A clean silver surface ($\phi = 4.7$ eV) is exposed to the ultraviolet light of wavelength 115 nm. What is the maximum velocity of photoelectrons emitted from this surface?
   Answer: $v_{max} = 1.08 \cdot 10^6$ m/s.

Case 2.

1. Write and explain the Einstein's formula for photoelectric effect.
2. Why is there photocurrent if the voltage between the photocathode and the anode is zero? (See Fig. 3.13.ii)
3. How can the maximum kinetic energy of the emitted electrons be determined?

4. *Problem.* When violet light with a wavelength of 400 nm falls on a clean cesium surface, the maximum velocity of photoelectrons is $6.5 \cdot 10^5$ m/s. What is the photoelectric threshold wavelength for this cesium surface?
   Answer: 640 nm.

Case 3.

1. Draw and explain the graph of volt-ampere characteristic, i.e. $I = f(U)$, for photoelectric effect. What is the photocurrent of saturation equal to?
2. Why does the velocity of photoelectrons depend on the incident light frequency?
3. What is called the threshold frequency?
4. *Problem.* When monochromatic light with a wavelength of 310 nm falls on a surface, the stopping potential necessary to stop emission of photoelectrons is 1.7 V. What is the work function for this surface?
   Answer: $\phi = 2.3$ eV.

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Охрімовський Андрій Михайлович
Подшивалова Оксана Володимирівна

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Національний аерокосмічний університет ім. М. Є. Жуковського
"Харківський авіаційний інститут"
61070, Харків-70, вул. Чкалова, 17
http://www.khai.edu
Видавничий центр "ХАІ"
61070, Харків-70, вул. Чкалова, 17
izdat@khai.edu

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